Title: p-adic analysis and zeta functions
C. Doug Haessig

Description: The course will be an introduction to the study of L-functions of exponential sums and zeta functions of algebraic varieties over finite fields using p-adic analysis. The course will be broken up into the following sections:
1. Basics of p-adic numbers and p-adic analysis
2. Introduction to L-functions and zeta functions
3. Trace formula and rationality
4. Degree estimates
5. p-adic cohomology theory
6. Dual theory (if time permits)
Texts:
1) Koblitz "p-adic numbers, p-adic analysis, and zeta functions"
2) Monsky "p-adic analysis and zeta functions"
3) Various papers in the literature
Prereq: Course will be kept mostly self-contained. Undergrad knowledge in algebra (some Galois but not much), linear algebra, analysis.
Learning Outcomes:
a) Basic understanding of p-adic numbers and p-adic analysis and how it may be used to lift questions from finite fields to characteristic 0.
b) Exposure to the Weil Conjectures on zeta functions, with a proof of the first conjecture, rationality of the zeta function.
c) An introduction to Dwork's pioneering work, including his p-adic analytic lifting of an additive character over a finite field.
d) Introduction to p-adic cohomology theory (de Rham type) in a self-contained way.
Schedule
Weeks 1 - 3: introduction to p-adic analysis
Weeks 4 - 6: introduction to p-adic Banach space theory
Weeks 7 - 9: Dwork's trace and the rationality of the zeta function
Week 10 - 12: Degree estimates of L-functions
Week 13 - 16: p-adic cohomology theory for L-functions