Proposal for a special topics graduate course "Monte Carlo Methods"

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This course has been offered as an Applied Math Special Topics course (Math 577) several times, taught by K. Lin and T. Kennedy, with enrollment regularly exceeding ten students. As one of the proponents of establishing this course as our regular offering, I am looking forward to carrying the torch this time around.

Introduction. Monte Carlo methods are among the most powerful and widely used computational tools in science and engineering. These methods combine ideas from multiple fields such as linear algebra, probability, stochastic processes, and spectral theory. In this course we will cover all the necessary mathematical foundations of Monte Carlo methods, explore the key problems and techniques related to their design and analysis.

Course prerequisites: basic (non-measure-theoretic) probability at the level of MATH464; linear algebra at the level of MATH313; ability to program using some general-purpose language at the level of *"read matrix A from a text file, compute $A^\dagger A$, produce a pdf containing histogram of its diagonal elements."* (More advanced techniques, such as various matrix factorizations, computation of eigenvalues and eigenvectors, Fourier transforms, etc will be covered in class as required, at least at the utility level.)

Course description. Pseudo-random number generators and tests for their quality; methods for sampling i.i.d. random variables with common distributions; methods for sampling jointly Gaussian random variables. Markov chains and Markov chain Monte Carlo; Metropolis-Hastings algorithm and error analysis; variance reduction; importance sampling. After studying the foundations, the students will be given projects to work on towards the end of the semester. These projects will cover a few specialized topics such as transition paths in multiscale systems and rare event simulation; variance reduction for stochastic differential equations; data assimilation, filtering, and parameter estimation.

Learning outcomes. Students will master the core principles of Monte Carlo algorithm design and analysis. They will be able to implement and apply Monte Carlo methods to solve typical problems arising in a variety of scientific and engineering applications.

Textbooks and notes. None required. Suitable general references at this level include:

- J. M. Handscomb and D. C. Hammersley, *Monte Carlo Methods,* Methuen 1965
- M. H. Kalos and P. A. Whitlock, *Monte Carlo Methods,* Wiley 2008
- J. S. Liu, *Monte Carlo Strategies in Scientific Computing,* Springer 2008
- A. B. Owen, *Monte Carlo theory, methods and examples,* 2013
- A. Asmussen and P. W. Glynn, *Stochastic Simulation: Algorithms and Analysis,* Springer 2007
- A. D. Sokal, *Monte Carlo methods in statistical mechanics: foundations and new algorithms,* NATO ASI Series, vol. 361

In addition, I will utilize lecture notes developed by T. Kennedy and K. Lin to assure core material consistency with previous years of instruction.