Title: Intro to stochastic pde
Instructor: Sunder Sethuraman
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Overview: There are a variety of stochastic pde's of interest. But, to be concrete, we will begin with versions of the stochastic heat equation, driven by white noise ξ of the form

$$\frac{\partial u}{dt} = \frac{\nu}{2}\frac{\partial^2 u}{\partial x^2} + b(u) + \sigma(u)\xi$$

in one dimension, where solutions $u = u(t,x)$ are well defined as random functions. We will also discuss higher dimensions, and other equations inspired from discrete approximations and physics motivations. Here, a white noise is a type of random field–a discrete approximation consists of i.i.d. random variables indexed by a lattice–that we will define, starting from basic notions.

The main idea of the course is understand some of the structure theories which allow to define and understand solutions of the stochastic pde's considered. The goal at the end is to approach current research on singular stochastic pde's, such as dynamical Φ^4 equation, KPZ equation, and parabolic Anderson models, whose solutions are distributions. The recent BAMS article by I. Corwin and H. Shen https://doi.org/10.1090/bull/1670 gives a summary of these stochastic pde's, a hot area these days.

References: We will follow the CBMS monograph by D. Khoshnevisan 'Analysis of Stochastic Partial Differential Equations' in the beginning. A more historical reference is the St. Fleur notes of J. Walsh. See also the Springer book 'A minicourse on stochastic partial differential equations' edited by D. Khoshnevisan and F. Rassoul-Agha. We will also supplement by notes of M. Gubinelli and N. Perkowski 'Lectures on singular stochastic pde's', and other sources.

Prerequisites: We will assume a previous course in graduate probability, at the level of 563. Otherwise, things will be covered, although previous experience with stochastic processes, stochastic (ordinary) differential equations, or partial differential equations will be helpful, but not required.

Approximate syllabus: The first half of the course will discuss how to solve the stochastic nonlinear heat equation above, and the associated material. In the second half, we will discuss other equations, motivated by discrete approximations in physical models, and also notions of how to solve them.

Expected learning outcomes: After the course, participants will be familiar with types of linear and nonlinear stochastic pde and how to solve them, as well as physics motivations. Participants will be ready to read research papers which involve stochastic pdes.

Questions are welcome: sethuram@math.arizona.edu or by stopping by.