MATH 105 (BAYLY) EXAM 3 SOLUTIONS (1)

(1)

	7	8	10	4
1st	A	B	C	A
2nd	B	C	A	C
3rd	C	A	B	B

(a) A has ~~more~~ 11 1st place votes, which is more than B (8) or C (10)

$\Rightarrow$ ATHENS is SIMPLE PLURALITY Winner.

(b) P with E: Eliminate B since fewest 1st place votes

& compare A vs C:

There are 11 that prefer A to C
18 prefer C to A

$\Rightarrow$ C is plurality with elimination winner

(c) Now it is

7	8	14
A	B	C
B	C	A
C	A	B

$\Rightarrow$ C is simple plurality winner.

(d) In P. with E., Eliminate A since A has fewest 1st place votes

NOW it is B with 15 versus C with 14

$\Rightarrow$ B is now Plurality with ELIMINATION winner

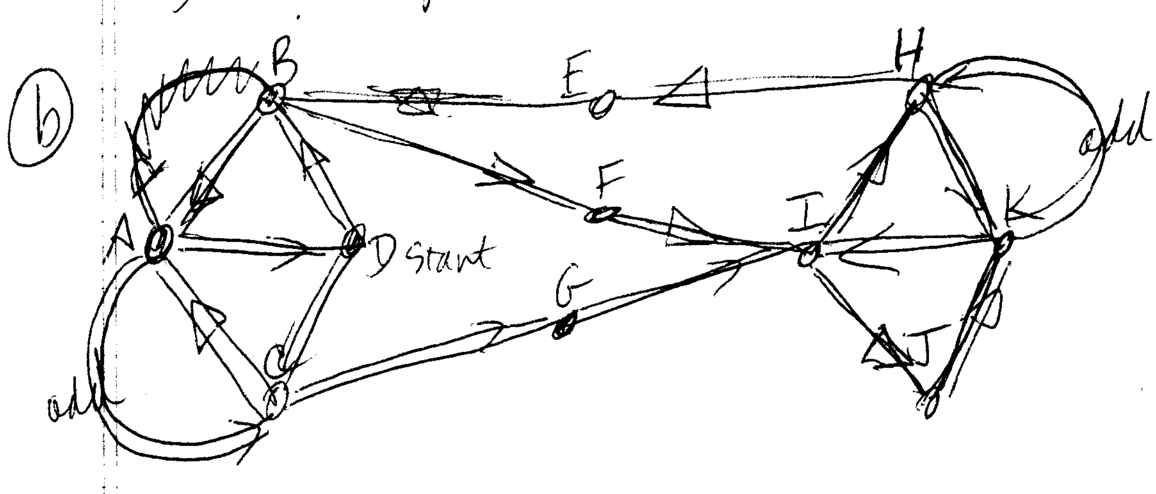
1d Continued: It's weird that B is now the winner, since the change in votes did not involve B! It's also weird that changing votes in favor of C made C go from a winner to a loser.

2 Recall that for an EULER CIRCUIT you can't have ANY odd nodes,

a and to have an EULER PATH you can only have TWO odd nodes (one at the start, one at the finish).

Here there are 6 ODD NODES!
Way too many! ⇒ no Euler path or circuit.

b



3

We added two curved ~~branches~~ edges to convert 4 of the odd nodes to even.

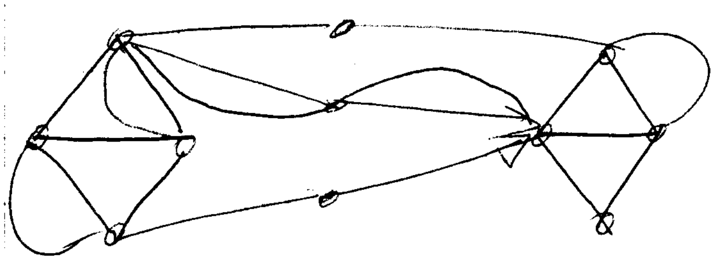
This leaves D and I as 2 odd nodes, and we can come up with a path that starts at one and finishes at the other. Explicitly

start D → B → A → C → A → D → C → G → I

J → K → H → K → I → H → E → B → F → I

Finish

(c) To get I & D to be even also, need to duplicate ~~B→A~~, D → B, B → F, and F → I



(3)

	8	7	6	2	1
1st	A	D	D	C	E
2nd	B	B	B	A	A
3rd	C	A	E	B	D
4th	D	C	C	D	B
5th	E	E	A	E	C

Total of 24 ballots

3) cont Since 13 people list D as their 1st

(a) choice, D is the majority winner.

(b) Borda count 1st = 5 to 5th = 1

A gets $8\times5 + 7\times3 + 6\times1 + 2\times4 + 1\times4 = 79$

B gets $8\times4 + 7\times4 + 6\times4 + 2\times3 + 1\times1 = 91$

C gets $8\times3 + 7\times2 + 6\times2 + 2\times5 + 1\times1 = 61$

D gets $8\times2 + 7\times5 + 6\times5 + 2\times2 + 1\times3 = 88$

E gets $8\times1 + 7\times1 + 6\times3 + 2\times1 + 1\times9 = 40$

91 is the biggest: B is Borda WINNER

despite D being a majority winner.

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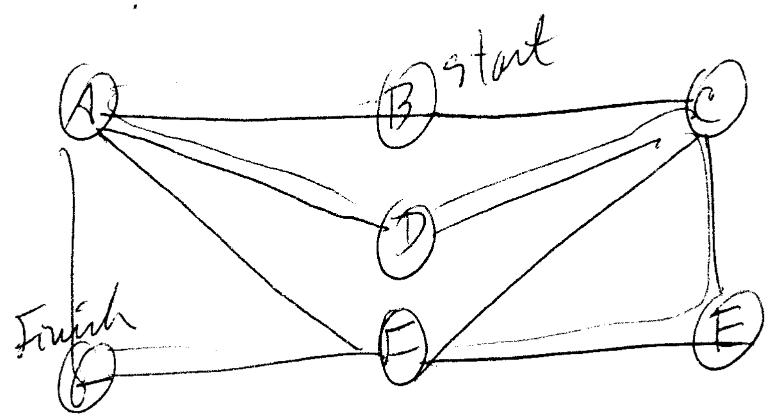
(4)

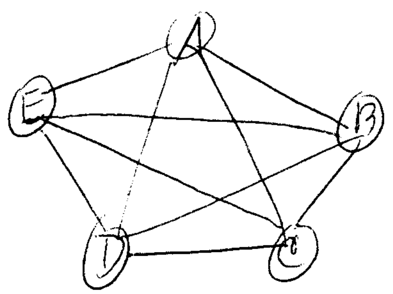


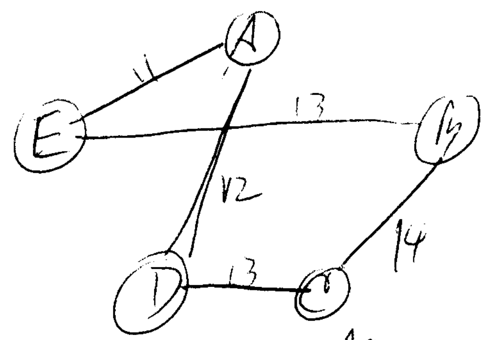
Hamiltonian path

$B \to A \to D \to C \to E \to F \to G$

4 cont What makes this difficult is that A & C are bottlenecks; once you go past them you can't get back to that side again.

5





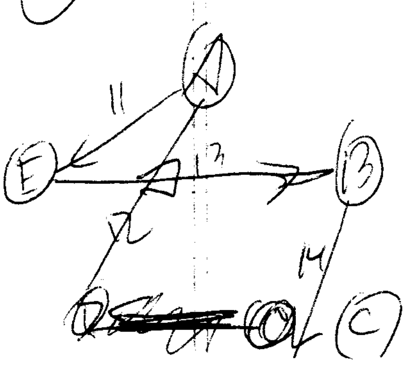
(a) Cheapest link: choose AE = 11, then AD = 12

then choose BOTH EB = 13 & CD = 13,

then FINALLY BC = 14 completes it.

TOTAL COST = 63 minutes

(b) Nearest neighbor starting at A ⇒ $A \xrightarrow{11} E \xrightarrow{13} B$



cost = 11 + 13 + 12 + a + 14 + 13 = 63

⇒ so a is the rate of change?

I have to get ~~crappy or~~: crazy or ?

(c) Neither cheapest link nor nearest neighbor is guaranteed to give shortest ~~path~~: route.
ONLY guaranteed way is to list all the routes & see how long each is.

NOT NEEDED but we have the space!

A B C D E A
15 + 14 + 13 + 22 + 11 = 75

A C B D E A
30 + 14 + 40 + 22 + 11 = 127

oh well, I'm tired!