Math 454, Home Work

(I) Answer the following questions for the equation

$$\frac{dx}{dt} = -x + y + x^2 + xy + x^3, \qquad \frac{dy}{dt} = x - y + x^2 + xy + y^3.$$

(a) Is this a one-zero eigenvalue case at $(0,0)$?
(b) Use a linear change of variables to turn this equation into standard form.
(c) Calculate the leading term of the center manifold.
(d) Determine the stability of $(0,0)$.

(II) Determine the stability of given equations at $(0,0)$ by first calculate the leading term of the center manifold.
(a) $\frac{dx}{dt} = -x^2 - y^2, \quad \frac{dy}{dt} = -y.$
(b) $\frac{dx}{dt} = x^3 - y^2, \quad \frac{dy}{dt} = -y + x^2.$
(c) $\frac{dx}{dt} = y, \quad \frac{dy}{dt} = -y + x^2.$

(III) Show that the equilibrium point at $(0,0)$ of the system

$$\frac{dx}{dt} = xy + ax^3 + bxy^2, \qquad \frac{dy}{dt} = -y + cx^2 + dx^2y$$

is (a) asymptotically stable if $a + c < 0$, and (b) asymptotically unstable if $a + c > 0$.

(IV) Consider the system

$$\frac{dx}{dt} = -x^3, \qquad \frac{dy}{dt} = -y + x^2.$$

(a) Write $h(x) = \sum_{n=1}^{\infty} h_n x^n$. Calculate h_n for all n.
(b) Determine the radius of convergence of $h(x)$ obtained in (a).
(c) Is $(0,0)$ a stable or unstable fix point?