Homework #4

Problem 1

Prove that for a general rectangular matrix A, null(A) is orthogonal to range(A^*) and range(A) is orthogonal to null(A^*)

Problem 2

The proof of Theorem 5.1 is very sketchy. Notice, for example, that the rank of the product AB of two matrices A and B of rank 2 does not have to be of rank 2. Example:

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \text{rank}(AB) = 1.$$

Please, give your own proof of Theorem 5.1. You cannot use theorems with larger numbers, i.e. 5.2, 5.3, and so on.

Problems from the textbook:

Problems 6.1, 6.2, 6.3, 6.4(a), 6.5, 7.1

Hint for problem 6.4: do not use the pseudoinverse $A(A^*A)^{-1}A^*$.