Questions for midterm (tentative, I may add more)

Some of the questions on midterm will be from this list, but some questions or problems not listed here will also be assigned.

Definitions:
Unitary matrix
All matrix and vector norms we studied
Projector
Complimentary projector
(Relative) condition number of a mapping
Condition number of a matrix
Floating point axioms ((13.5) and (13.7))
Stability of an algorithm (if we cover it before midterm)
Backward stability (if we cover it before midterm)

Things to derive or prove:
range$(A)^{\perp} \perp$ null(A^*)
Inverse of an upper-triangular matrix is upper-triangular
Eigenvalues of a Hermitian matrix are real, and eigenvectors corresponding to distinct eigenvalues are orthogonal
Norm inequalities, problem 3.3.
Show that $\forall A$

$$A = U \begin{pmatrix} \sigma_1 & 0 \\ 0 & B \end{pmatrix} V^*$$

with U, V unitary, and $\sigma_1 = ||A||_2$ (this is one step of SVD construction).

Thms 5.1, 5.4, 5.5, 5.8, 6.1.
range$(I - P) =$ null(P)
range$(P) \cap$ null$(P) = \{0\}$
Derive a formula for projecting on span$(q_1, ..., q_n)$ where $q_1, ..., q_n$ form an orthonormal set.

Prove that a shortest distance (i.e. 2-norm) between a vector b and some subspace V not containing b is along the vector orthogonal to V; in other words, if $y \in V$ minimizes the distance form b to V, then y is unique, and $(y - b)$ is orthogonal to V.

Derive a formula for projecting orthogonally on range(A), where A is arbitrary, without use of SVD or QR decomposition (see p.46).

Problems 6.3, 6.5.
Describe Gram-Schmidt orthogonalization and count the flops; same for the modified GS.
Describe Householder triangularization and count the flops

Explain how one can solve a least squares problem using (a) QR decomposition (b) SVD decomposition (c) system of normal equations

(continues on the next page)

Condition number of a matrix-vector multiplication (derive the formula)
Condition number of a system of equations (derive the formula)

The discrete Fourier transform can be represented as multiplication by a matrix Q. Give a formula for the entries of Q, and, by direct computation, show that these columns form an orthogonal set (see my notes on the DFT)

Suppose matrix A has an SVD $A = U\Sigma V^*$. Suppose also that there is a vector $\vec{w}$ such that $\vec{w}$ is not parallel to $\vec{v}_1$, $||\vec{w}||_2 = 1$ and $||A\vec{w}||_2 = \sigma_1$. Prove that $\sigma_2 = \sigma_1$.