Homework #6 (Complete)

Problem #1

Design an implicit collocation-based two stage Gauss-like method with $c_1 = 1/4$ and $c_2 = 3/4$. Present your method in the form

$$\begin{array}{c|c} c & A \\ \hline & b^T \end{array}$$

Problem #2

Problem 4.8(a,b,c), p. 111, (do not forget to justify your answers). Hint for (b): mapping $\frac{1+az}{1+bz}$ maps y-axis onto a circle.

Problem #3

Design an Adams-Moulton (implicit) method using uniformly spaced points t_n, t_{n-1}, and t_{n-2}, and a BDF method using uniformly spaced points t_n, t_{n-1}, t_{n-2}, and t_{n-3}. Since the answers to these problems are well known, you should present details of your computations (otherwise - no credit).

Problem #4

Find the error in the form $C_{p+1}h^p y^{(p+1)}(t_n) + \mathcal{O}(h^{p+1})$ for the second BDF method in the table 5.3. Show all details of your work for full credit. (Find p and C_{p+1}).

Problem #5

Write a MATLAB program that finds the coefficient C_l for any multistep method given by an array of α's and an array of β's. Find the error term in the form $\approx C_{p+1}h^p y^{(p+1)}(t_n)$ for the first five BDF methods in the table 5.3. (It's enough to find p and C_{p+1} for each method). **Hint**: Use MATLAB function **rats(...)** to represent your output as rational numbers.

Problem #6

Consider the second Adams-Bashforth method (Table 5.1). Analyze its region of absolute stability by considering only real non-positive values of $z = \lambda h$. Assuming that on the real line this region is represented by a single interval, show that the endpoints of this interval are -1 and 0.

Problem #7

Consider the second BDF method (Table 5.3). Consider only real non-positive values of $z = \lambda h$, and show that all the roots of the equation

$$\rho(\xi) = z\sigma(\xi)$$

converge to 0 in the limit $z \to -\infty$. Explain, why this implies that the method has the stiff decay property.