Study guide for the midterm (I may add topics later...)

Most of the questions on midterm will be from this list, but questions
and problems not listed here can also be assigned.

Definitions and things to know without a proof:

1. Lipschitz condition
2. Functional iteration
3. Linear, quadratic, and cubic convergence
4. Simple iteration (chord method), Newton iteration, method of false position, in $\mathbb{R}^1$ and in $\mathbb{R}^n$.
5. Geometric interpretation of the simple iteration, Newton iteration, and method of false position, in $\mathbb{R}^1$.
6. Norm and a semi-norm (p. 177). Examples
7. Weierstrass thm, p. 183
8. Modulos of continuity of a function
9. Bernstein polynomials, simple properties, thm 1, p. 185.
10. Lagrange interpolation polynomial
11. Vandermonde matrix
12. Runge's counterexample
13. Osculating interpolation (definition)
14. Inner product and inner-product-induced norm (definitions and properties)
15. Fourier series as orthogonal projection on an orthonormal set
16. divided differences, forward differences, central differences
17. Orthogonal polynomials (Legendre, Chebyshev - how are they constructed?)
18. Orthogonal functions, orthogonal functions with respect to a weight
19. Gram-Schnidt process for functions
20. Least square problems
21. Least square problems with weight
22. Thm 5, p.204; Thm 7, p. 206; Thm 8, p. 209; and Thm 9; p. 211, without proofs.
23. Newton interpolating polynomial. Divided differences.
24. Newton interpolating polynomial with equispaced points. Forward differences.
25. Gauss form of interpolating polynomial, centered differences.
26. Quadrature rule, nodes, weights, degree of accuracy.
27. Gauss quadrature

Things to prove or derive:

1. Thm 1, p. 86, Corollary, p. 88.
2. Thm 2, p. 90, Corollary, p. 90.
3. Show that if $g'(\alpha) = 0$ and $g''(\alpha) \neq 0$, simple iteration results in a quadratic convergence (under additional conditions; derive a sufficient condition for convergence in this case) (p. 94)
4. Show that the Newton's method is a second order method (for $\mathbb{R}^1$ version of the method)
5. Thm 1, p. 110. Please be ready to show all the details of the proof (a simple reference to previous theorems is not enough).
6. Thm 2, p. 111, and subsequent stuff (p. 112).
7. Show that the Newton's method is a second order method for $\mathbb{R}^n$ version of the method (see p.114).
8. Prove that Vandermonde matrix is non-singular
9. Prove uniquness of interpolating polynomial (of degree n through $n+1$ distinct points).
10. Thm 1, p.190.
11. Prove Cauchy-Schwartz inequality for functions
12. Pythagorean thm, Bessel's inequality, Parseval's equality for functions.
13. Least squares problem for functions: show that the orthogonal projection on $span\{g_1(x), ..., g_n(x)\}$ (where g's are an orthonormal set of functions) minimizes $||f - \sum_{k=1}^{n} \alpha_k g_k||_2$.
14. Give a (reasonably) tight estimate for the convergence of the Fourier series of the k-times differentiable function.
15. Prove that, given a sequence of orthonormal polynomials, the generalized Fourier series (in these polynomials) of a function $f(x)$ converges to $f(x)$ in L_2 sense.
16. Proofs for Fourier interpolating series and Chebyshev polynomials that were parts of Homework #2.
17. Derive formulas 1(c), (4), (6), pp. 246-248.
18. Derive formulas (4), (5), (6), pp. 261-262.
19. Prove that divided differences are invariant to permutation of points
20. Thm 1, p. 249
21. Thm 1, p. 289
22. Thm 3, p. 316
23. Prove that Gaussian quadrature with n nodes has degree of precision $2n-1$. (This is Thm 1 p. 328 in one direction; see also Embree's lecture notes, p. 134-136. You can use without the proof the fact that the zeros of orthogonal polynomials are all single and real, and lie within the interval of orthogonality).
24. Thm 2, p.328.