Homework #1

Problem 1

(a) Prove that the following system has a unique solution:

$$\begin{cases} 30x_1 - 10 + 10\cos(0.1x_2 + 1) & = 0 \\ 20x_2 + 15 + 5\sin(0.1x_1 + 0.1x_3) & = 0 \\ 70x_3 - 30 + 5\mathrm{e}^{-x_1^2} & = 0 \end{cases}.$$

(b) Write a program in Matlab that finds the solution of this system by a simple iteration with $g(x) = x - Af(x)$ where A is either a constant or a constant matrix (your choice), and $f(x)$ is the left hand side of the above system. The initial guess should be $x^{(0)} = (1, 1, 1)$ and the iterations should be stopped when the condition $||f(x^{(n)})||_2 \leq 10^{-6}$ is satisfied.

Please, print out the number of iterations required to reach the desired accuracy. Submit the code and the printout.

Hint: Components of $f(x)$ can be conveniently programmed in Matlab using inline functions, i.e.

```
F1= @(x)(30 * x(1) - 10 + 10 * cos(0.1 * x(2) + 1));
...
F= @(x)[f1(x), f2(x), f3(x)];
```

so that Y=F(X) takes vector x and returns vector $y = f(x)$.

Problem 2

(a) Write a program in Matlab that finds the solution of the system below by a simple iteration with $g(x) = x - Af(x)$ where A is either a constant or a constant matrix (your choice), and $f(x)$ is the left hand side of the system:

$$\begin{cases} x_1 - x_2 - 10 + 10\cos(0.1x_1) & = 0 \\ 2x_1 + x_3 - 20 + 10\sin(0.1x_2) & = 0 \\ x_1 - 0.1x_2 + x_3 - 30 + 10\mathrm{e}^{-x_3^2} & = 0 \end{cases}.$$

The initial guess should be $x^{(0)} = (1, 1, 1)$ and the iterations should be stopped when the condition $||f(x^{(n)})||_2 \leq 10^{-6}$ is satisfied. You do not have to prove anything, just find A that makes the iteration converge. Please, print out the number of iterations required to reach the desired accuracy. Submit the code and the printout.

(b) Numerically solve the system from part (a) using the Newton's method, with the same initial guess and stopping criterion. Please, print out the number of iterations required to reach the desired accuracy. Submit the code and the printout.

Hint 1. In the problem 2 you are allowed to use the INV(...) function from Matlab, but no other high level functions (like NORM(...)anywhere else, please.

Hint 2. In all problems, please execute FORMAT LONG before running the programs, so that all numbers are printed with lots of digits.