Homework #5

(All the stability results in this homework should be obtained directly, without reference to Section 4.4 where the general theory is presented.)

Problem #1

Find the region of stability for the implicit midpoint method, and for the implicit trapezoid method. Show that they have the same region of stability.

Problem #2

Show that the explicit trapezoid method and the explicit midpoint method have the same region of stability. Find the interval of stability corresponding to real values of λ.

Problem #3

Find the boundary of the region of stability for the explicit trapezoid method. This will require some programming. In particular, the problem reduces to finding all points in the complex plane where the absolute value of some quadratic polynomial $P(z)$ equals to one. This is equivalent to $P(z) = \exp(i\theta)$ for some θ. For any given θ the above equation is a quadratic equation in z and can be easily solved, yielding solution(s) $z(\theta)$. If one plots all $z(\theta)$ for all values of $\theta \in [0, 2\pi]$, the resulting curve will be the boundary of the region. (Don't forget to submit the figure and the code).