

Math 250a (Fall '07) - Homework 9 extra problems

1. For each of the following differential equations, before you plot the slope field first determine which of the three possible symmetries the slope field has and what this tells you about the solutions. Then plot the slope field and some solutions to check your answer. (The three possible symmetries are reflection in the x-axis, reflection in the y-axis and reflection in the origin.)

(a) $y^3 y' = x^2$

(b) $y' = \frac{y}{2x}$

(c) $y' = \sin(xy)$

2. Consider the dif eq

$$\frac{dy}{dx} = f(x)h(y)$$

For each of the following cases determine which of the three possible symmetries the slope field has and what this tells you about the solutions. Check your answer by making up and f and h for each case and plotting the slope field.

(a) f and h are both even functions

(b) f and h are both odd functions

(c) f is an odd function and h is an even function

(d) f is an even function and h is an odd function

3. Suppose that Y is a solution to the differential equation

$$\frac{dY}{dx} = Y^2(1 - Y) \tag{1}$$

Now consider the differential equation

$$\frac{dy}{dx} = 2y^2(5 - y) \tag{2}$$

By considering functions of the form $aY(bx)$ where a and b are constants, express $y(x)$ in terms of $Y(x)$.

4. The differential equation

$$\frac{du}{dx} = (1 + u)(2 + u)$$

can be solved by the usual method and partial fractions. Luckily for you, I have already done this and found that the solution with $u(0) = 0$ is given by

$$u(x) = 2 \frac{e^x - 1}{2 - e^x}$$

Now let y be the solution of

$$\frac{dy}{dx} = (c + y)(2c + y)$$

with $y(0) = 0$. The parameter c is positive. Find $y(x)$. Hint : try $y(x) = au(bx)$.