10/1

1

Example

$$\frac{dy}{dx} = y(1-y^2) = y - y^3$$

Equilibria: $y = 0, +1, -1$

$$g(y) = y(1-y^2)$$

$$g'(y) = (1-y^2) + y(-2y)$$

$$= 1 - y^2 \cancel{+} - 2y^2$$

$$= 1 - 3y^2$$

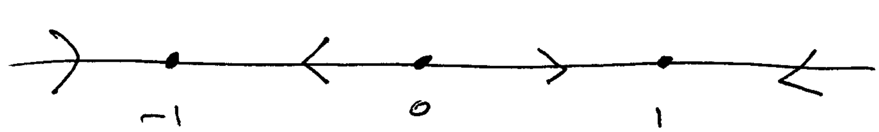
$$g'(0) = 1$$

$$g'(1) = 1 - 3 = -2$$

$$g'(-1) = -2$$

0 unstable

1, -1 stable



10/1 2

$$\frac{d^2y}{dx^2} = ?$$

$y = y(x)$

$$\frac{d^2y}{dx^2} = 1 \cdot \frac{dy}{dx} - 3y^2 \frac{dy}{dx}$$

$$= (1-3y^2)\frac{dy}{dx}$$

$$= (1-3y^2)\, y\, (1-y^2)$$

§ 2.6 Bifurcation Diagrams

Example

$$y' = y(a-y)$$

a is parameter

Eq. solutions : $y = 0,\ a$

$$g(y) = ay - y^2$$

$$g'(y) = a - 2y$$

$$g'(0) = a$$

$$g'(a) = a - 2a = -a$$

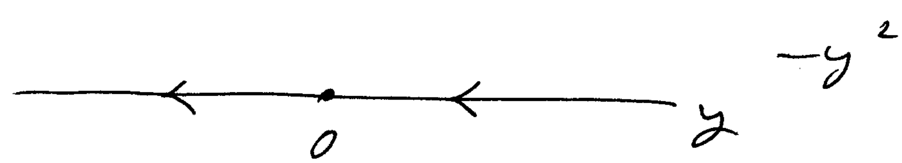
$a > 0$ $\quad$ 0 unstable

$\quad\quad\quad\;\;$ a stable

$a < 0$ $\quad$ 0 stable

$\quad\quad\quad\;\;$ a unstable

$a = 0$

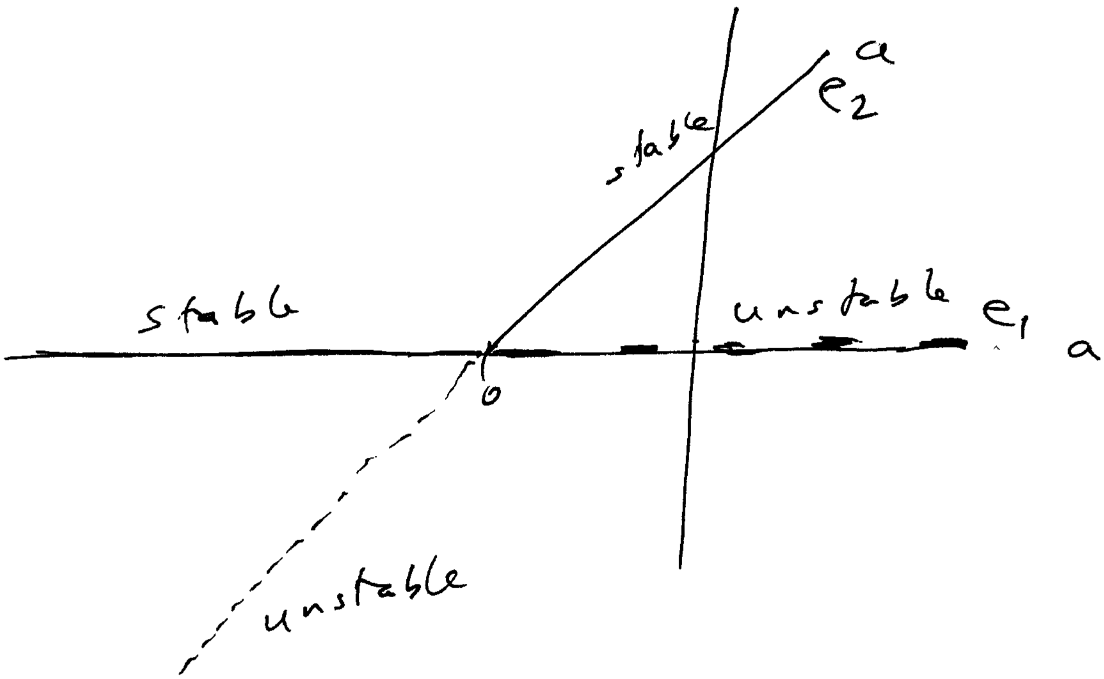


semistable

Bifurcation diagram

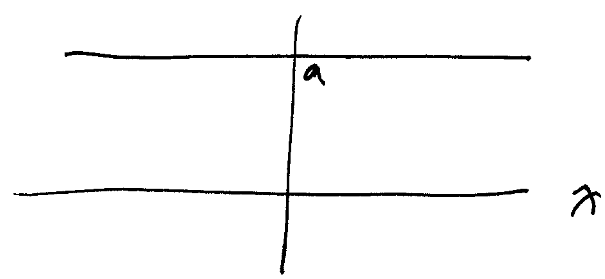
— stable

- - - unstable



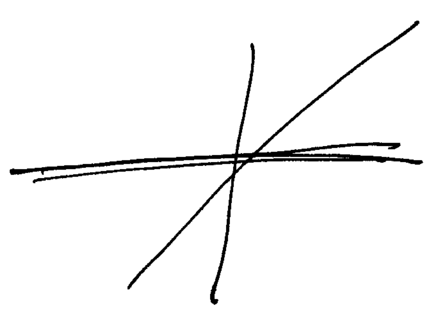
$$\left.\begin{aligned} e_1(a) &= 0 \\ e_2(a) &= a \end{aligned}\right\}\quad \text{equilibria}$$

$$y(x) = a$$



$$f_1(x) = 0$$

$$f_2(x) = x$$



10/1 (5)

$$\frac{dx}{dt} = x\left(e^{x^2} - a\right)$$

Equilibria: $x = 0$ always

$$e^{x^2} - a = 0$$

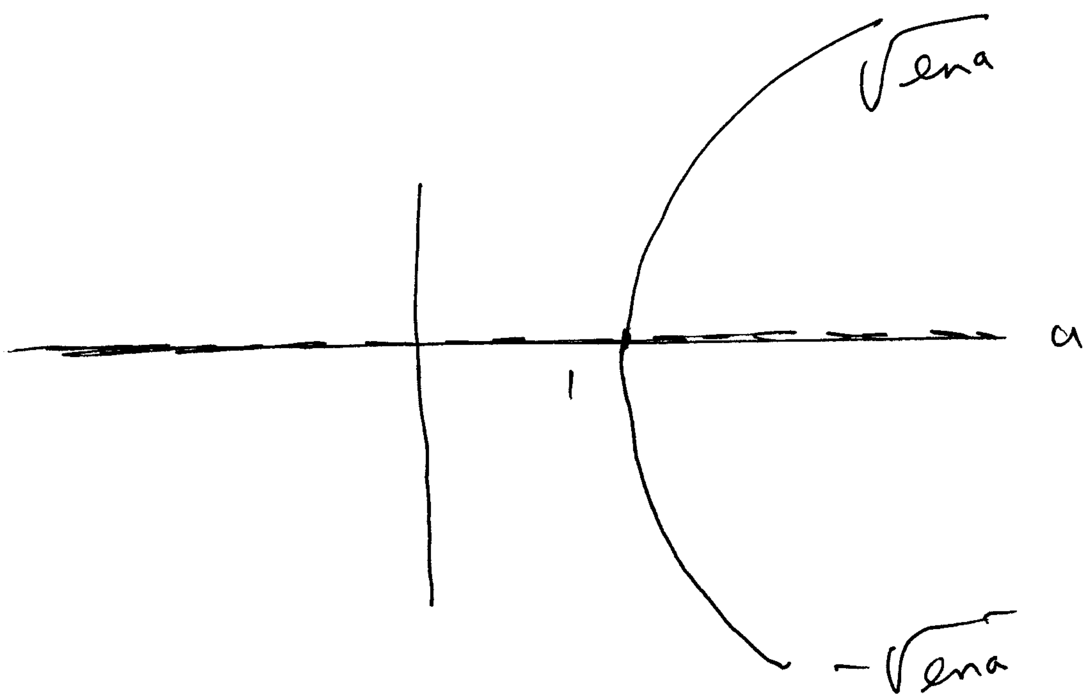
$$e^{x^2} = a$$

$$x^2 = \ln a \quad \text{if } a > 0$$

$$x = \pm\sqrt{\ln a} \quad \text{if } a > 1$$

$a < 1$ 1 eq. at 0

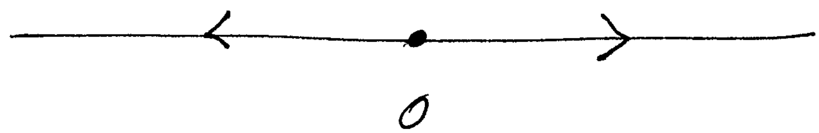
$a > 1$ 3 eq. : $0, \pm\sqrt{\ln a}$



10/1

6

$a < 1$ $\qquad \frac{dy}{dx} = x(e^{x^2} - a)$



unstable

$a > 1$