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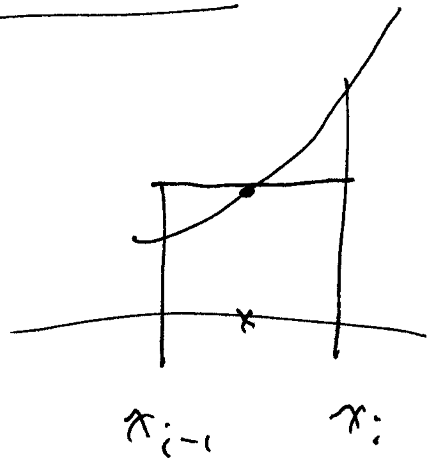
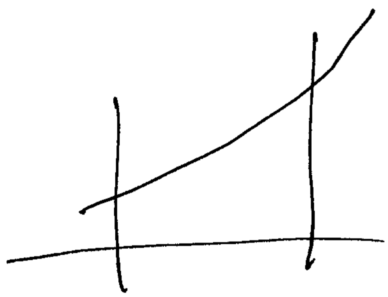
1

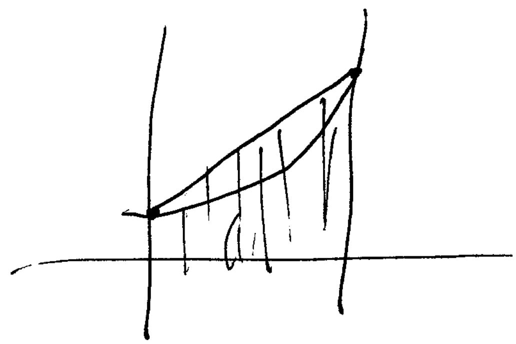
midpoint



$$\Delta x \; \underbrace{f\left(\frac{x_{i-1}+x_i}{2}\right)}_{\text{midpoint}}$$

Trapezoid





Area of trapezoid

$$\Delta x \; \frac{f(x_{i-1}) + f(x_i)}{2}$$

EXAM Fri Oct 19

Through 3.2 in dif eq.

No 7.5 from calc. on exam

$$\text{Error} = \left| \int_a^b f(x)\,dx - \text{approximation} \right|$$

~~Usually~~ Always Error $\to 0$ as $N \to \infty$.

If Error goes as $\dfrac{c}{N^p}$

we say the method has

order p.

Example

$$\int_1^2 (x - 2x^3) \ln x \, dx$$

$$= \frac{9}{8} - 6 \ln 2$$

LEFT $p = 1$ RIGHT $p = 1$

$N = 10, 20, 40, 80$

$$\frac{c}{N} = \frac{c}{10}, \frac{c}{20}, \frac{c}{40}, \frac{c}{80}$$

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3

If $p = 2$

$N = 10, 20, 40$

$$\text{err} = \frac{c}{(10)^2}, \frac{c}{(20)^2}, \frac{c}{(40)^2}$$

error goes down by factor of 4 when we double N.

Midpoint : $p = 2$

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4

Simpson $p = 4$

$$\frac{C}{N^4}$$

Bounds

f decreasing   LEFT is over estimate
RIGHT is under

f increasing   LEFT is under
RIGHT is over

f concave up   TRAP is ~~under~~ over

concave down   TRAP is under

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Midpoint

f concave up

Red area = midpoint

f concave up mid is under

concave down over