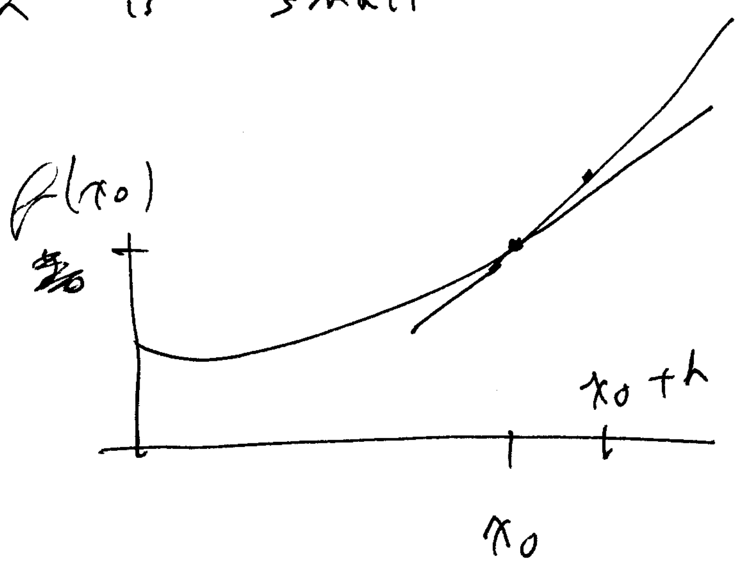
§ 3.3 Numerical methods

$$\frac{dy}{dx} = g(x, y), \quad y(x_0) = y_0$$

Tangent line approx

$$f(x_0 + h) \approx f(x_0) + f'(x_0)\,h$$

h is small



Euler's method

h = step size

Approximate $y(x)$ at

$x_0,\ x_0 + h,\ x_0 + 2h,\ \ldots,\ x_0 + nh$

Know $y_0 = y(x_0)$

$x_k = x_0 + kh$

$$y_1 = y(x_0 + h)$$

$$\simeq y(x_0) + y'(x_0)\, h$$

$$= y(x_0) + g(x_0, y(x_0))\, h$$

$$= y_0 + g(x_0, y_0)\, h$$

$$y_2 = y(x_0 + 2h) = y(x_1 + h)$$

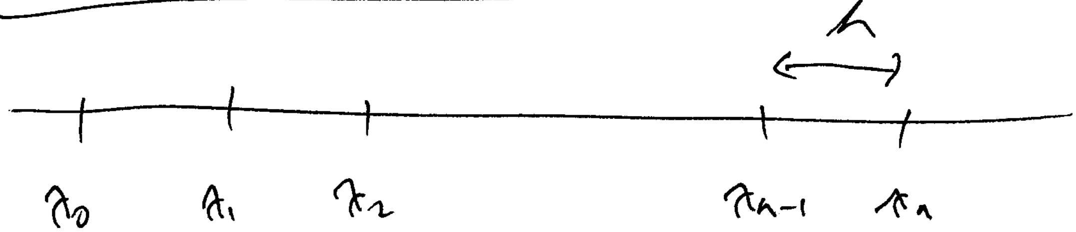
$$= y(x_1) + y'(x_1)\, h$$

$$= y_1 + g(x_1, y_1)\, h$$

$$y_n = y(x_n) = y(x_{n-1} + h)$$

$$\simeq y(x_{n-1}) + y'(x_{n-1})\, h$$

$$\boxed{y_n = y(x_{n-1}) + g(x_{n-1}, y_{n-1})\, h}$$



Example

$$\frac{dy}{dx} = \frac{x}{y}, \qquad y(1) = 2$$

Take $h = .1$

n	x_n	y_n	$g(x_n, y_n)$
0	1	2	.5
1	1.1	2.05	.5366
2	1.2	2.1037	.5704
3	1.3	2.16607	

$$y_1 = y_0 + g(x_0, y_0)\,h = 2 + .5 \cdot .1$$

$$y_2 = y_1 + g(x_1, y_1)\,h = 2.05 + .5366 \cdot .1$$