3.3 Continued

10/26

1

Order

Solve the dif eq. over some interval $a \le x \le b$.

error $=$ $y(b)$ $-$ approximation

$y(a)$ $\leftarrow$ initial condition

Typically

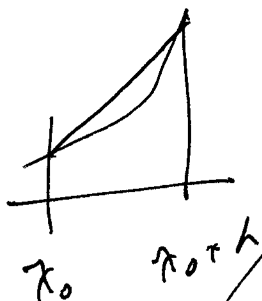
$$\text{error} \approx c h^{p}$$

Euler $p = 1$ first order

Better methods

FTC:

$$y(x_0+h) = \int_{x_0}^{x_0+h} y'(x)\,dx + y(x_0)$$



$$= \int_{x_0}^{x_0+h} g(x,y)\,dx + y(x_0)$$

$$\approx g(x_0, y(x_0))\,h + y(x_0)$$

LEFT corresponds to Euler

what about trapezoid

$$\approx \frac{h}{2}\left[g(x_0, y(x_0)) + g(x_0+h, \underline{y(x_0+h)})\right] + y(x_0)$$

?

Approximate $y(x_0+h) \approx \cancel{y(x_0) + g(x_0}$

$$y(x_0) + g(x_0, y(x_0))\,h$$

Let $m_0 = g(x_0, y_0)$

$$k_1 = g(x_1, y_0 + g(x_0,y_0)\,h)$$

$$y(x_1) = y(x_0) + \tfrac{1}{2} h (m_0 + k_1)$$

Modified Euler, Heun's method $p=2$

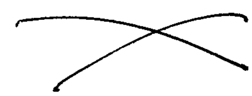
Even better method

Runge - Kutta fourth order

$p = 4$

Numerical problems

If $g(x,y)$ is nice then solution curves ~~doesn't~~ don't cross

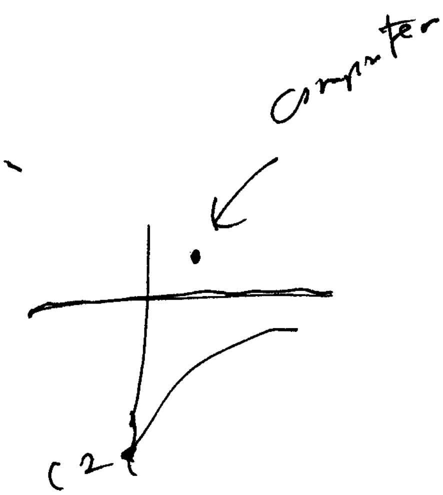


Computer doesn't know this.

$$\frac{dy}{dx} = y^2 \qquad y(0) = -12$$

$h = 0.1$ Euler

$y = 0$ is a solution



$$y(0.1) = y(0) + g(0,-12)\,0.1$$

$$= -12 + 144 \cdot 0.1$$

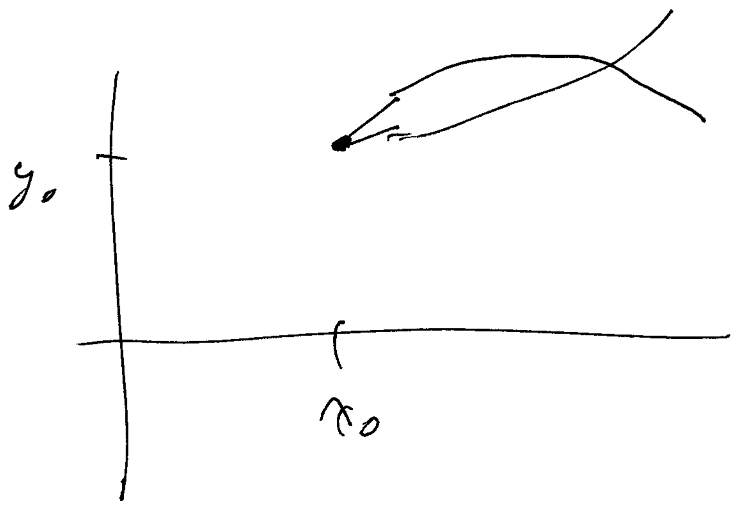
$$= 2.4$$

10/26

(4)

3.4 Comparing Solutions

Book's statement of theorem is slightly wrong.

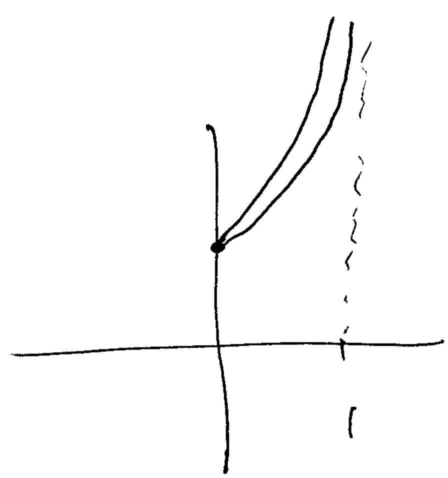


not allowed

Example

$y' = y^2 \qquad y(0) = 1$

$y = \dfrac{1}{1-x}$ by sep. of vars



Look at

$y' = y^2 + x$

10/26

5

$f(x,y) = y^2 + x$

$g(x,y) = y^2$

$f(x,y) > g(x,y)$ for $x > 0$

So theorem says solution to

$\frac{dy}{dx} = f(x,y)$

with initial cond. $y(0) = 1$

is above solution to

$\frac{dy}{dx} = g(x,y)$ with same I.C.

So solution to $\frac{dy}{dx} = y^2 + x$

must have a vertical asymptote

at $x = a$, $a \le 1$.