10/3
(1)

2.3 #1

$$\frac{dy}{dx} = a\,y(b-y)$$

$$y(0) = 35$$

$$\rightarrow y(1) = 700$$

$$b = 35{,}000$$

$$y(x) = \frac{b y_0}{(b-y_0)e^{-abx} + y_0}$$

a unknown

$y_0 = y(0) = 35$

$$700 = \frac{35{,}000 \times 35}{(35{,}000 - 35)e^{-a\,35{,}000} + 35}$$

$$y(x) = .9 \times 35{,}000$$

Solve for x

2.3
#3

10/3
(2)

$$y' = a y (b - y) \leftarrow$$

$$y(x) = \frac{1}{u(x)}$$

$$y'(x) = \frac{-1}{u(x)^2} u'(x)$$

$$-\frac{1}{u(x)^2} u'(x) = a \frac{1}{u(x)} \left(b - \frac{1}{u(x)}\right)$$

$$-u' = a(bu - 1)$$

$$u' = a(1 - bu)$$

$$\frac{du}{dx} = a(1 - bu)$$

$$\frac{du}{1 - bu} = a\,dx$$

$$-\frac{1}{b} \ln|1 - bu| = ax + C$$

~~#1~~ 2.3 #7

10/3
(3)

$$\frac{dy}{dx} = \frac{1}{2}(1-y^2)$$

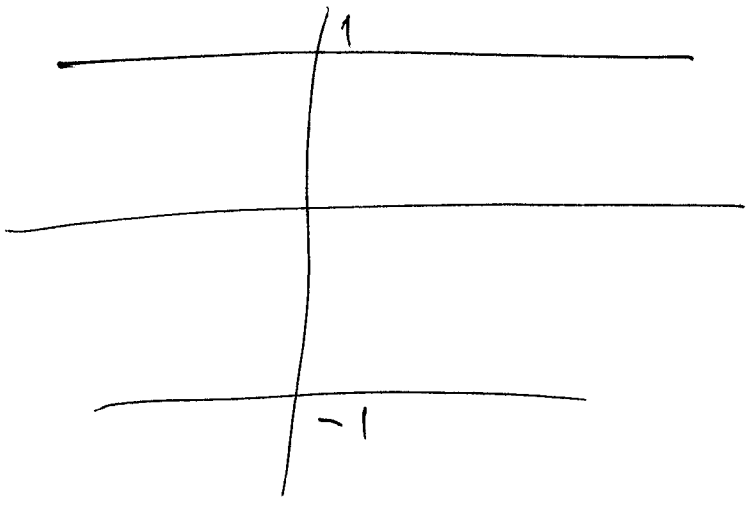
$$2\frac{dy}{1-y^2} = dx$$

$$\int 2\frac{dy}{1-y^2} = \int \frac{2\,dy}{(1-y)(1+y)}$$

$$= \int \left(\frac{A}{1-y} + \frac{B}{1+y}\right) dy$$

$$= \int \left(\frac{1}{1-y} + \frac{1}{1+y}\right) dy$$

$$\ln\frac{1+y}{1-y} = x + C$$



2.4
#6

$$\frac{dy}{dx} = a y(b-y)$$

10/3

(4)

$$y(x) = \frac{b y_0}{(b-y_0)e^{-abx} + y_0}$$

Suppose $y_0 > b$

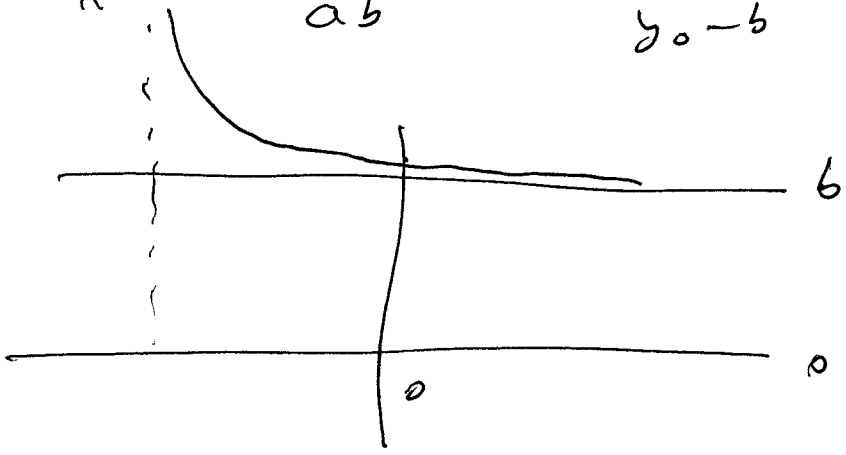
$$(b-y_0)e^{-abx} + y_0 = 0$$

$$e^{-abx} = \frac{-y_0}{b-y_0}$$

$$= \frac{y_0}{y_0 - b} > 1$$

$$-abx = \ln \frac{y_0}{y_0 - b} > 0$$

$$x = \frac{-1}{ab} \ln \frac{y_0}{y_0 - b} < 0$$

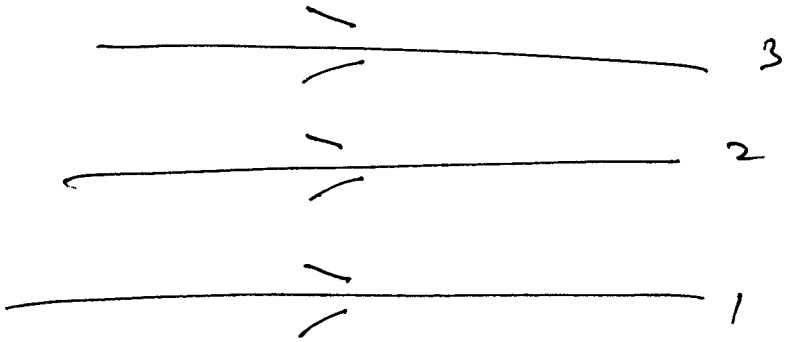


2.4

(19) (a)

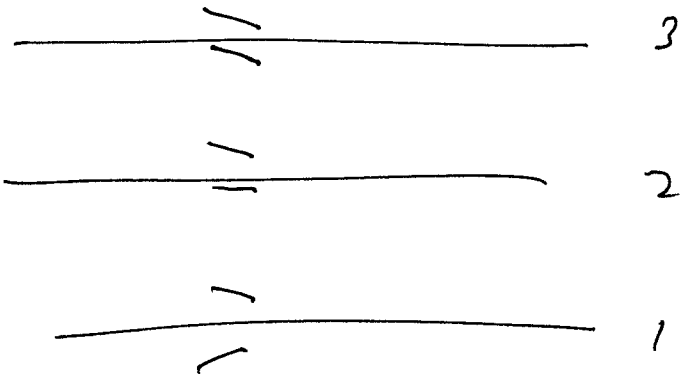
10/3 (5)

$y = 1, 2, 3$ all stable

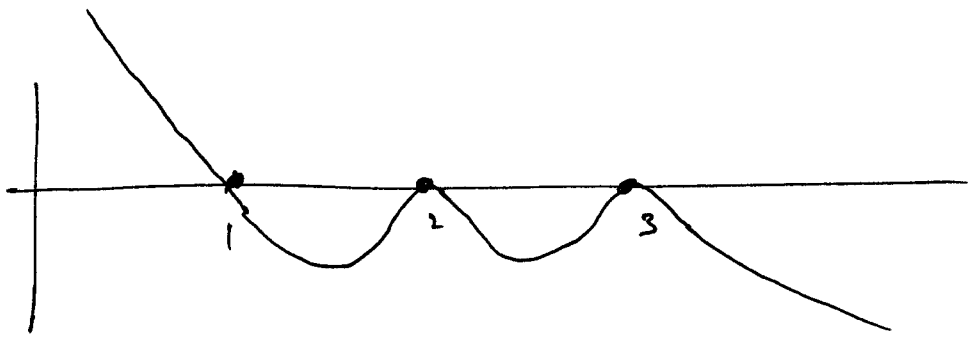


(c) 1 stable

2, 3 semistable



$(y-1)(y-2)^2(y-3)^2$



PDP Problem

10/3
6

$$\frac{dy}{dt} = ay(b-y) - c$$

$$aby - ay^2 - c = 0$$

$$-ay^2 + aby - c = 0$$

$$\frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$\frac{-ab \pm \sqrt{a^2b^2 - 4(-a)(-c)}}{2(-a)}$$

$$\frac{ab \pm \sqrt{a^2b^2 - 4ac}}{2a}$$

two equi. if $a^2b^2 - 4ac > 0$

no equi if $a^2b^2 - 4ac < 0$

$$a^2b^2 = 4ac$$

$$c = \frac{a^2b^2}{4a} = \frac{ab^2}{4}$$

$$\geq b$$