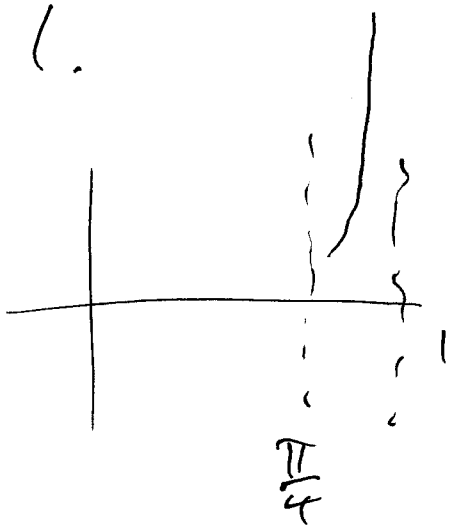
3.4 #1

$x \geq 0$

$$\frac{dy}{dx} = y^2 + x^n, \quad y(0) = 1$$

Show vertical asymptote between $\frac{\pi}{4}$ and 1.

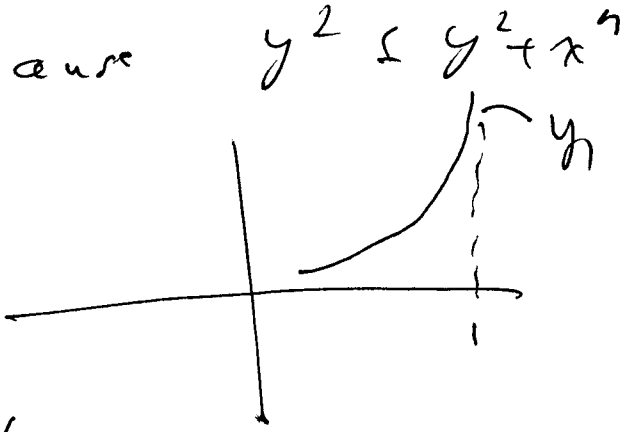


Consider

$$\frac{dy_1}{dx} = y_1^2, \quad y_1(0) = 1$$

Then $y_1 \leq y$ because $y^2 \leq y^2 + x^n$

$$y_1 = \frac{1}{1-x}$$



$\Rightarrow$ y has vertical asymptote at $x = 1$ or before.

$$\frac{dy}{dx} = y^2 + x^n \qquad n > 1$$

for $0 \le x \le 1$, $x^n \le 1$

$$\frac{dy_2}{dx} = y^2 + 1 \qquad y(0) = 1$$

Comp. Thm says

$y_2 \ge y$ for $0 \le x \le 1$

Solve $\frac{dy_2}{dx} = y^2 + 1$, $y(0) = 1$

$$y = \tan(x + C) = \tan\left(x + \frac{\pi}{4}\right)$$

Vertical asymp at $x = \frac{\pi}{4}$.

$\Rightarrow$ vert. asymp of y is at $x = \frac{\pi}{4}$ or later.

4.3 Modelling

Example Car is going 60 mph. It coasts to a stop.

t (sec)	v (mph)	$\frac{dv}{dt}$ $(h=5)$
0	60	
5	52.2	-1.39
10	46.1	-1.08
15	41.4	-.86
20	37.5	-.71
25	34.3	-.59
30	31.6	

Goal: Find a d.e. eq. for $v(t)$

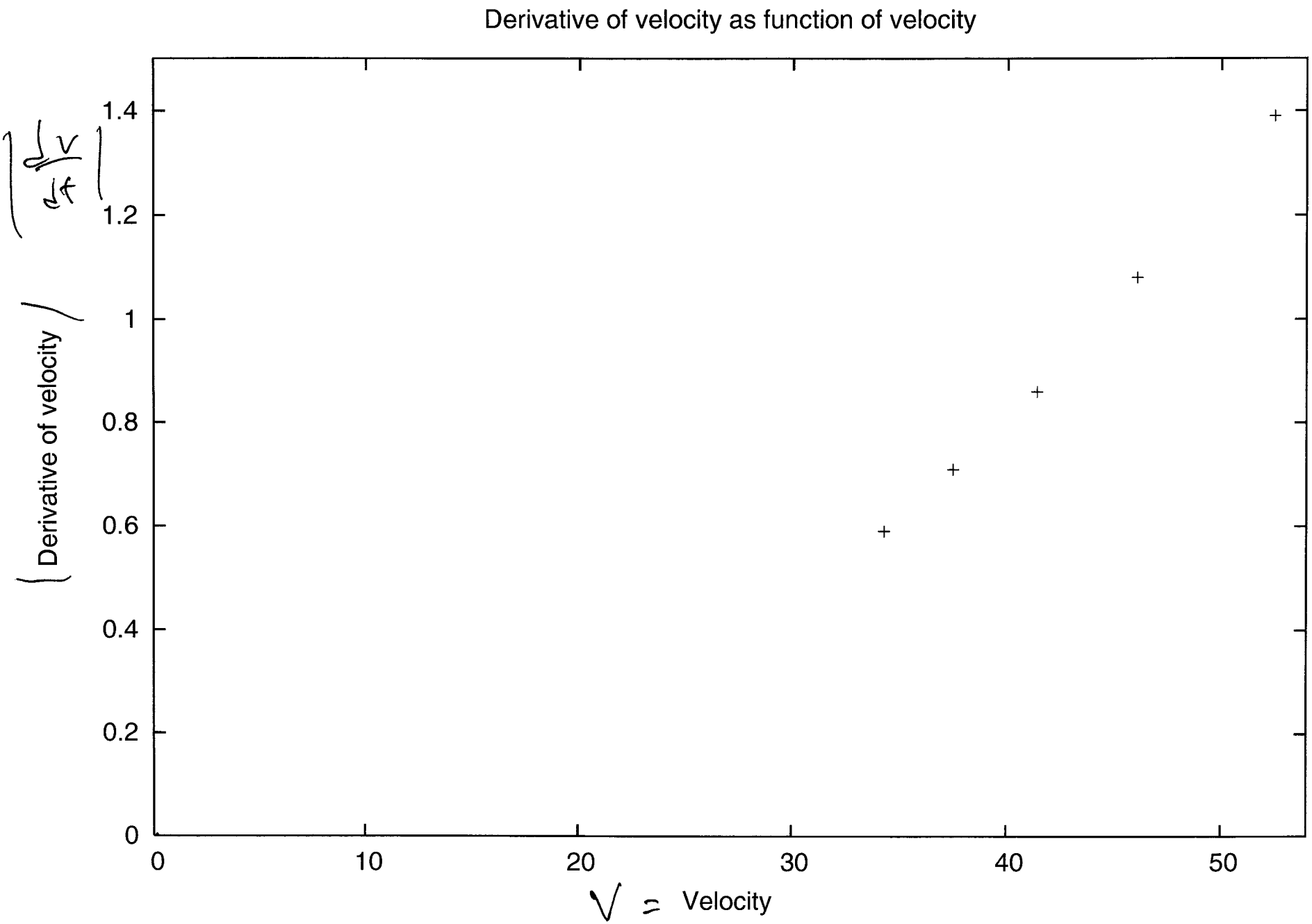
$v(t)$ = velocity.

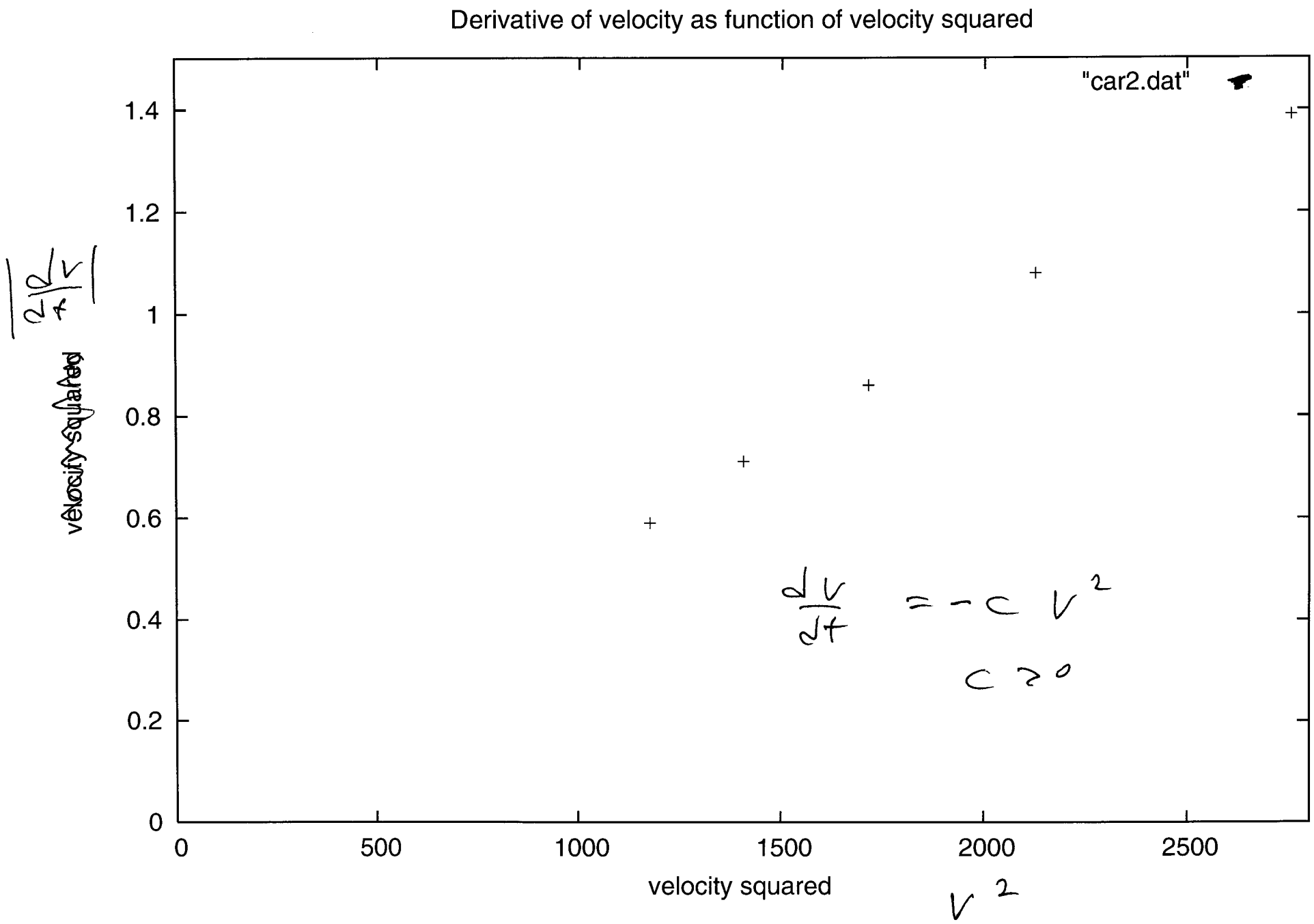
$\frac{dv}{dt}$ depends on v ?

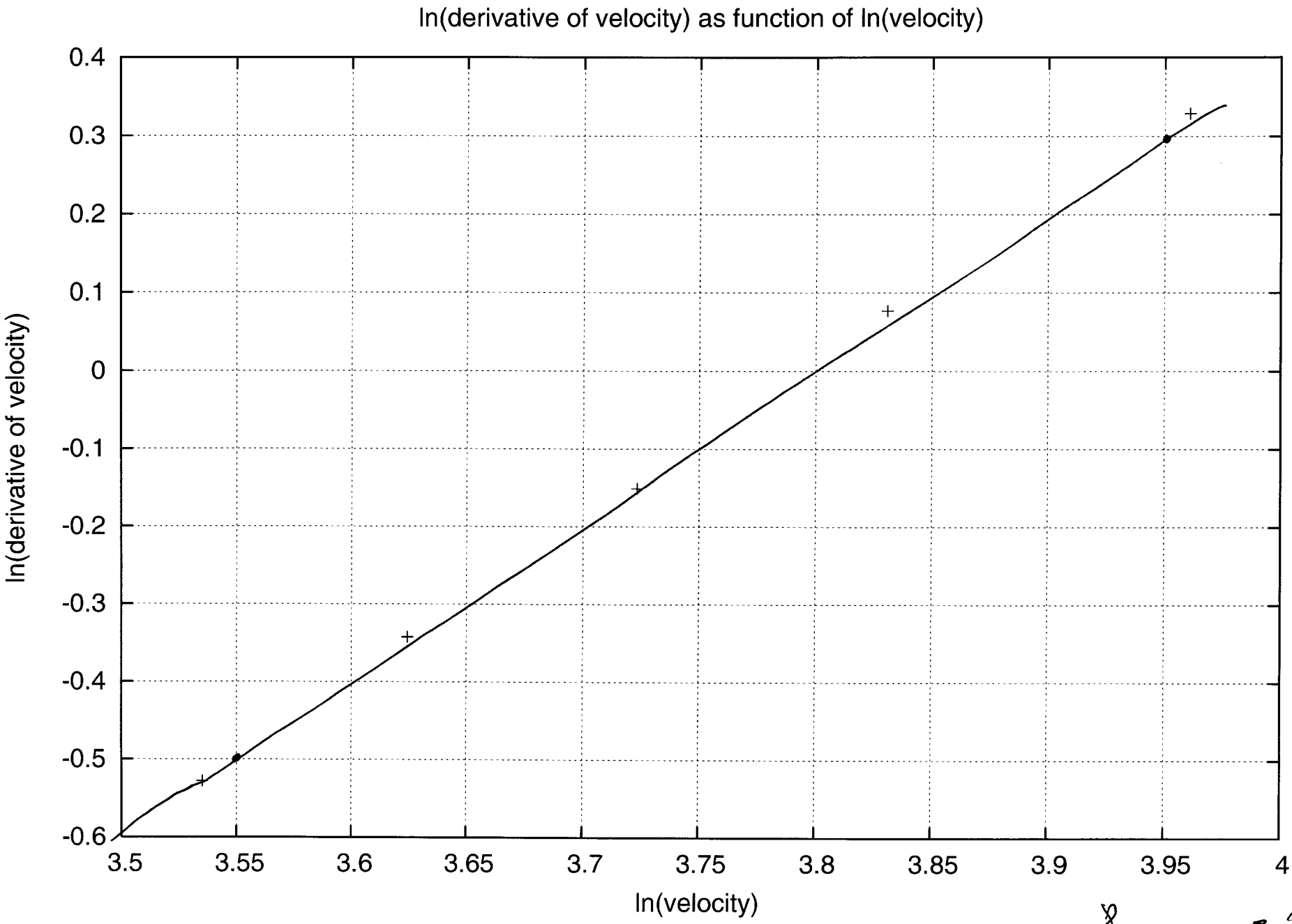
Possibilities $\frac{dv}{dt} = \text{const}$

$$\frac{dv}{dt} = -cv \qquad c > 0$$

$$\frac{dv}{dt} = -cv^p$$


Derivative of velocity as function of velocity
Derivative of velocity
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
10
20
30
40
50
Velocity


Derivative of velocity as function of velocity squared
"car2.dat"
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
500
1000
1500
2000
2500
velocity squared
velocity squared
$\frac{dv}{dt} = -c\,v^2$
$c > 0$
v^2


ln(derivative of velocity) as function of ln(velocity)
ln(derivative of velocity)
ln(velocity)
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
3.5
3.55
3.6
3.65
3.7
3.75
3.8
3.85
3.9
3.95
4

$\frac{.8}{.4} = 2$