

Chap 1

$$\frac{dy}{dx} = g(x)$$

$$y(x) = \int g(x) dx$$

10/5
①

Chap 2

$$\frac{dy}{dx} = g(y)$$

$$\int \frac{dy}{g(y)} = \int dx$$

Chap 3

$$\frac{dy}{dx}$$

$$= g(x, y)$$

~~$\frac{dy}{dx} = g(x, y)$~~

No analytic solutions in general

$$\int \frac{dy}{R(y)} = \int \frac{dx}{f(x)}$$

if

$$g(x, y) = f(x)h(y)$$

$$\frac{dy}{dx} = f(x, y)$$

10/5


If so line:

$$f(x, y) = m$$

$$f(x, y) = 0$$



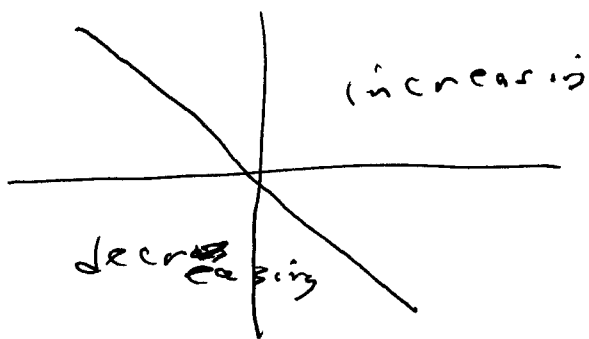
possible local
 max's, min's
 of solution

$$y' = x + y$$

$$y(x) = c$$

increases vs decreases

increase if $x + y > 0$



$$x + y = 0$$

$$y = -x$$

(0, 1) satisfies

$$x + y > 0$$

Concavity

$$y'' = 1 + y' = 1 + x + y$$

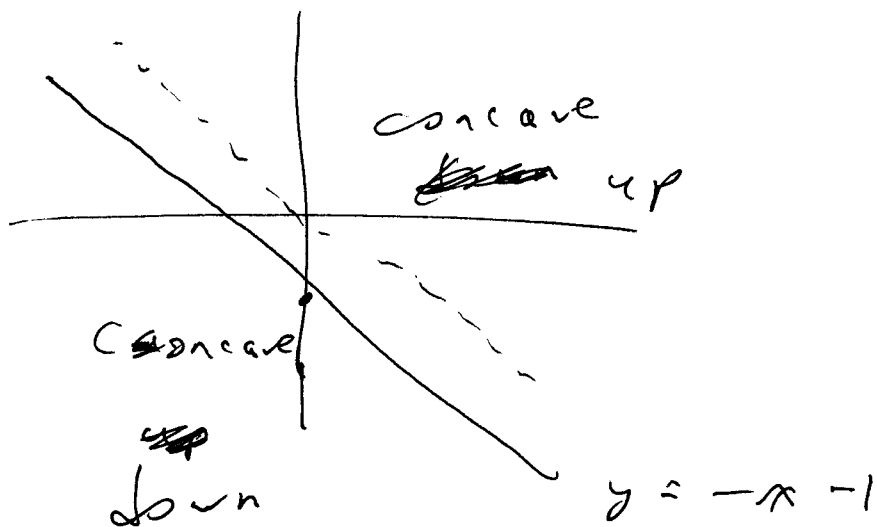
Example

$$y'' = 1 + x + y$$

60/5
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$$1 + x + y = 0$$

$$y = -x - 1$$



Looks like $y = -x - 1$ is a solution. Any it:

$$y' = -1$$

$$x + y = x + (-x - 1) = -1$$

So yes, it satisfies

$$y' = x + y$$

$$\frac{dy}{dx} = g(x, y)$$

$y(x)$

Example

$$\frac{dy}{dx} = x(1-y^2)$$

10/5
②

Equilibrium

$$y(x) = c$$

$$0 = x(1-c^2)$$

$$c = \pm 1$$

So $y(x) = +1$, $y(x) = -1$
are equilibria.

Monotonicity

$$x > 0, -1 < y < 1$$

increase

$$x < 0, -1 < y < 1$$

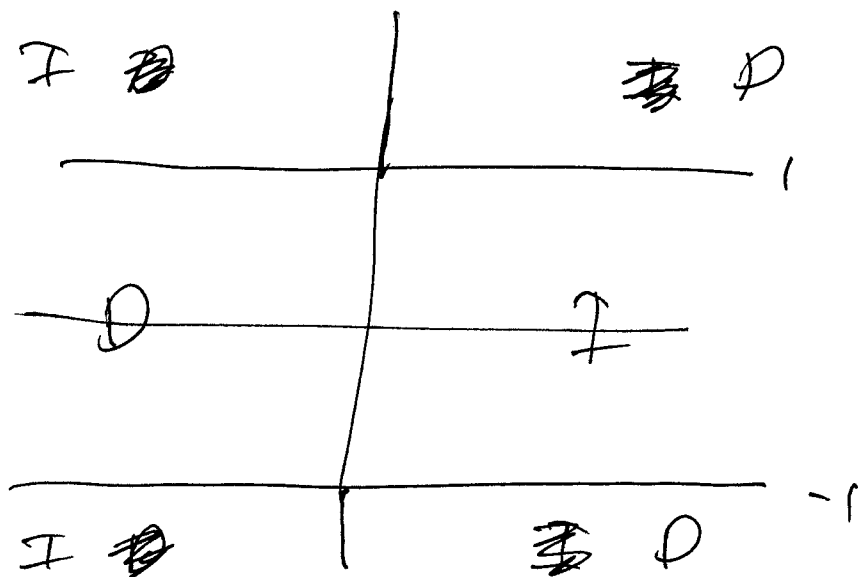
decrease

$$x > 0, |y| > 1$$

decrease

$$x < 0, |y| > 1$$

increase



$$\frac{dy}{dx} = x(1-y^2)$$

10 (5)
⑤

$$\frac{d^2y}{dx^2} = (1-y^2) + x \frac{d}{dx}(1-y^2)$$

$$= (1-y^2) + x(-2)y \frac{dy}{dx}$$

$$= 1-y^2 - 2xy \cdot x(1-y^2)$$

$$= (1-y^2) [1 - 2x^2y]$$

$$\frac{d^2y}{dx^2} = 0 \quad y = \pm 1$$

$$1 = 2x^2y$$

$$y = \frac{1}{2x^2}$$

$$y = 1, \quad y = -1, \quad y = \frac{1}{2x^2}$$

are boundaries of regions where
it is concave up, concave down