3.1 Cont $y(x)$

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$$(x+y)^3 + (x-y)^3 = 1$$

Graph this curve.

Idea: $\frac{d}{dx}$ it and get a dif. eq. to study.

$$3(x+y)^2\left(1+\frac{dy}{dx}\right) + 3(x-y)^2\left(1-\frac{dy}{dx}\right) = 0$$

$$\left[(x+y)^2 - (x-y)^2\right]\frac{dy}{dx}$$

$$+ (x+y)^2 + (x-y)^2 = 0$$

$$4xy\frac{dy}{dx} + 2x^2 + 2y^2 = 0$$

$$\frac{dy}{dx} = -\left(\frac{x^2+y^2}{2xy}\right)$$

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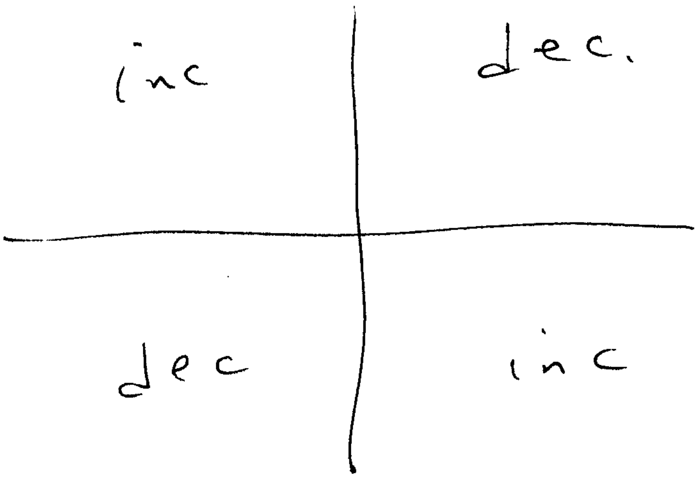


monotonicity

$x = 0 \qquad y^3 - y^3 = 1$

$y = 0 \qquad x^3 + x^3 = 1$

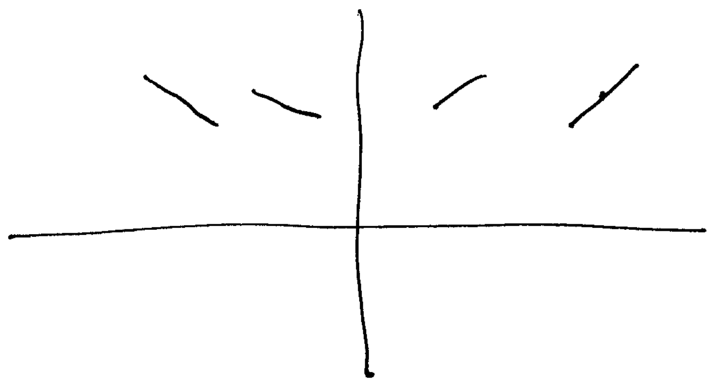
$x^3 = \frac{1}{2}$

$x = \left(\frac{1}{2}\right)^{1/3}, \quad y = 0$

$y\left(\left(\frac{1}{2}\right)^{1/3}\right) = 0$

§ 3.2 Symmetry, Scaling

Suppose $g(-x, y) = -g(x, y)$



Let $y(x)$ be a solution.

Define $\bar{y}(x) = y(-x)$

Then $\frac{d}{dx} \bar{y}(x) = \frac{d}{dx} y(-x)$

$$= y'(-x)(-1) = -y'(-x)$$

But $y'(x) = g(x, y)$

So $\bar{y}'(x) = -g(-x, y(-x))$

$$= g(x, y(-x)) = g(x, \bar{y})$$

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So $\bar{y}(x) = y(-x)$ is another solution.

$$\bar{y}(0) = y(0)$$

So $\bar{y}, y$ have same initial condition at $x = 0$.

So by uniqueness thm

$$\bar{y}(x) = y(x)$$

i.e. $y(-x) = y(x)$

i.e. y is even.

Example ~~$g(x,y)$~~

$$\frac{dy}{dx} = x \cos y = g(x,y)$$

$$g(-x,y) = -x \cos y = -g(x,y)$$

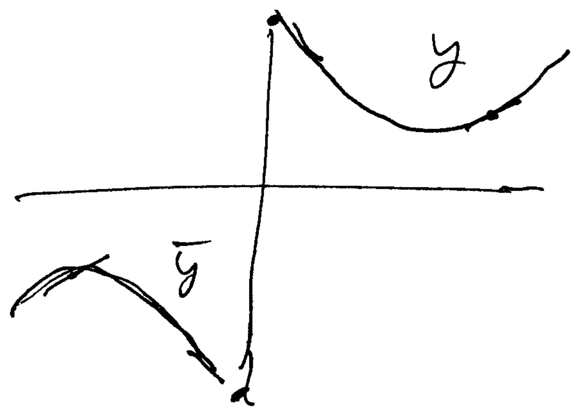
$\therefore$ f satisfies condition

$$\frac{dy}{dx} = \sin(xy)$$

$$\sin(-xy) = -\sin(xy)$$

Now suppose

$$g(-x, -y) = g(x, y) \quad (\star)$$



$$\frac{dy}{dx} = g(x, y)$$

Let $y(x)$ be a solution.

Let $\bar{y}(x) = -y(-x)$

$$\frac{d}{dx}\bar{y}(x) = -y'(-x)(-1) = y'(-x)$$

$$= g(-x, y(-x))$$

$\star$ says $g(-x, y) = g(x, -y)$

So $\frac{d}{dx}\bar{y}(x) = g(+x, -y(-x))$

$$= g(x, \bar{y}(x))$$

$\bar{y}(x)$ is another solution.

$\bar{y}(0) = -y(0)$

Scalings

Suppose $Y(x)$ is a solution of

$$Y' = Y(1-Y) \quad \Big| \quad Y'(x) = Y(x)(1-Y(x)),$$

~~Let $y(x) = cY(dx)$~~ , ~~c, d constants~~

Let $y(x) = aY(bx)$, a, b constants

$$y'(x) = a\,Y'(bx)\,b$$

$$= ab\,Y(bx)\,(1-Y(bx))$$

$$= b\,y(x)\left(1 - \frac{1}{a}y(x)\right)$$

$$= \frac{b}{a}\,y(x)\,(a - y(x))$$

$$\frac{dy}{dx} = A\,y\,(B-y)$$

y = population $\qquad x$ = time

change units thousand of people

time are secs → mins, hours

3.1 # 2 Given

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$$y' = g(y + ax + b)$$

Look at

$$u(x) = y(x) + ax + b = \bar{y}(x)$$

$$u'(x) = y'(x) + a$$

$$= g(y + ax + b) + a$$

$$= g(u(x)) + a$$

$$u' = g(u) + a$$

$$\frac{du}{dx} = g(u) + a$$

Fri Oct 19

Second Exam