11/16

1

§ 5.2

$$\frac{dx}{dt} = 1 - \frac{2x}{t+200}, \quad x(t) = \text{concentration}$$

$$\frac{dx}{dt} + \underbrace{\frac{2}{t+200}}_{p(t)} x = 1$$

$$\int p(t) = 2 \ln(t+200)$$

$$\text{int. factor} = \exp(2 \ln(t+200))$$
$$= (t+200)^2$$

$$(t+200)^2 \left[\frac{dx}{dt} + \frac{2}{t+200} x\right] = (t+200)^2$$

$$\frac{d}{dt}\left[x (t+200)^2\right] = (t+200)^2$$

$$x(t+200)^2 = \int (t+200)^2 \, dt$$

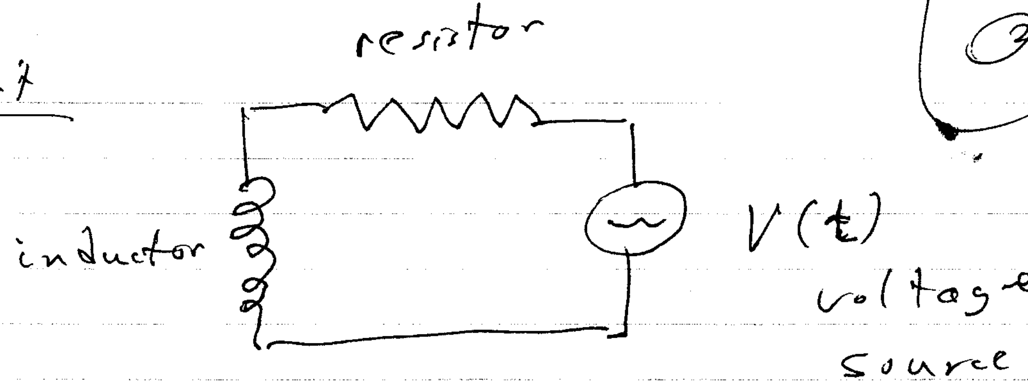
$$x(t+200)^2 = \frac{1}{3}(t+200)^3 + C$$

$$x = \frac{1}{3}(t+200) + \frac{C}{(t+200)^2}$$

11/16

2

Circuit



$$L \frac{dI}{dt} + R I = V(t)$$

L = inductance

R = resistance

$V(t)$ = voltage soure

$I(t)$ = current $\leftarrow$ unknown

$$\frac{dI}{dt} + \frac{R}{L} I = \frac{V(t)}{L}$$

$$p = \frac{R}{L} \qquad \int p \, dt = \frac{R}{L} t$$

$$\text{I.F.} = e^{\frac{R}{L} t}$$

(1) $$V(t) = \begin{cases} 0 & t < 0 \\ V_0 & t > 0 \end{cases} \quad \text{constant}$$

$$e^{\frac{R}{L}t}\left(I' + \frac{R}{L} I\right) = \frac{V_0}{L} e^{\frac{R}{L}t}$$

$$\left(e^{\frac{R}{L}t} I\right)' = \frac{V_0}{L} e^{\frac{R}{L}t}$$

11/16

3

$$L \frac{dI}{dt} + R I = v(t)$$

$$V(t) = \begin{cases} 0 & t < 0 \\ V_0 \cos(t) & t > 0 \end{cases}$$

$$\left(e^{\frac{R}{L}t} I\right)' = e^{\frac{R}{L}t} V(t)$$

$$= e^{\frac{R}{L}t} V_0 \cos(t)$$

$$e^{\frac{R}{L}t} I = V_0 \int e^{\frac{R}{L}t} \cos(t)\, dt$$

$$\int = e^{\frac{R}{L}t} \sin t + \frac{R}{L} e^{\frac{R}{L}t} \cos t$$

$$- \frac{R^2}{L^2} \int$$

$$\int e^{\frac{R}{L}t} \cos t \, dt = \frac{e^{\frac{R}{L}t}\left(\sin t + \frac{R}{L}\cos t\right)}{1 + \frac{R^2}{L^2}}$$

$$e^{\frac{R}{L}t} I(t) = \frac{V_0}{1 + \frac{R^2}{L^2}} \left(e^{\frac{R}{L}t}\right)\left(\sin t + \frac{R}{L}\cos t\right)$$

$$+ C$$

$$I(t) = \frac{V_0}{1 + \frac{R^2}{L^2}} \left(\sin t + \frac{R}{L}\cos t\right) + C e^{-\frac{R}{L}t}$$

11/16

④

$$e^{\frac{R}{L}t} I = \frac{V_0}{L} \int e^{\frac{R}{L}t}$$

$$= \frac{V_0}{L}\left(\frac{L}{R} e^{\frac{R}{L}t} + C\right)$$

$$I(t) = \frac{V_0}{R} + C\frac{V_0}{L} e^{-\frac{R}{L}t}$$

$$I(0) = 0$$

~~steady state~~

$$0 = \frac{V_0}{R} + C\frac{V_0}{L}$$

$$C = -\frac{L}{R}$$

steady state

$$I(t) = \frac{V_0}{R} - \frac{V_0}{R} e^{-\frac{R}{L}t}$$

transient

As $t \to \infty$ $\quad I(t) \to \frac{V_0}{R}$

11/16

5

$I(0) = 0$

$$I(0) = \frac{V_0}{1 + \frac{R^2}{L^2}} \frac{R}{L} + C$$

transient part $= C e^{-\frac{R}{L} t}$

$$\text{Steady state} = \underbrace{\frac{V_0}{1 + \frac{R^2}{L^2}} \left(\sin t + \frac{R}{L} \cos t\right)}_{\text{periodic} \quad 2\pi}$$

Newton heating

At 8 am. it's $70°$ outside

my house is $70°$

Outside temperature $T_{out}(t)$

$$T_{out}(t) = 70 + 5t$$

$t =$ # hours past 8. a.m.

At 10 am. my house is $75°$

No A/C $\quad T(t) =$ temp in house

What is temp in house at 2 p.m.

$$\frac{dT}{dt} = k\left[-T(t) + T_{out}(t)\right]$$

$$\frac{dT}{dt} = -kT + k(70 + 5t)$$

11/16 ⑥

$$\frac{dT}{dt} + kT = k\,(70 + 5t)$$

$p = k \qquad \int p\, dt = kt$

$$\text{I.F.} = e^{kt}$$

$$\left(e^{kt}\, T\right)' = k(70 + 5t)\, e^{kt}$$

$$e^{kt}\, T = \int k\,(70 + 5t)\, e^{kt}\, dt$$

$$= 70k \int e^{kt}\, dt + 5k \int t\, e^{kt}\, dt$$

$$= 70k\, \frac{1}{k} e^{kt} + 5k \frac{d}{dk} \int e^{kt}\, dt$$

$$= 70\, e^{kt} + 5k \frac{d}{dk} \frac{1}{k} e^{kt} + C$$

$$= 70\, e^{kt} + 5k\, \frac{t}{k} e^{kt} + 5k\left(\frac{-1}{k^2}\right) e^{kt} + C$$

$$= 70\, e^{kt} + 5t\, e^{kt} - \frac{5}{k} e^{kt} + C$$

$$T(t) = 70 + 5t - \frac{5}{k} + C\, e^{-kt}$$

$T(0) = 70 \qquad T(2) = 75$

$T(6) = ?$