

Extra office hour  
Tues 3:30 - 4:30

11/19  
①

3c.  
5.2

$$\frac{1}{7} \int e^{7x - 49x^2}$$

$$y' - 7y = 14t$$

$$y(0) = 0$$

$$e^{-5t} = e^{-7t}$$

$$e^{-7t} (y' - 7y) = e^{-7t} 14t$$

$$(e^{-7t} y)' = e^{-7t} 14t$$

$$e^{-7t} y = 14 \int e^{-7t} t dt$$

$$u = t \quad dv = e^{-7t}$$

$$du = dt \quad v = -\frac{1}{7} e^{-7t}$$

$$-\frac{1}{7} t e^{-7t} + \frac{1}{7} \int e^{-7t} dt$$

5.2 #7

$$k = -0.2$$

11/29  
②

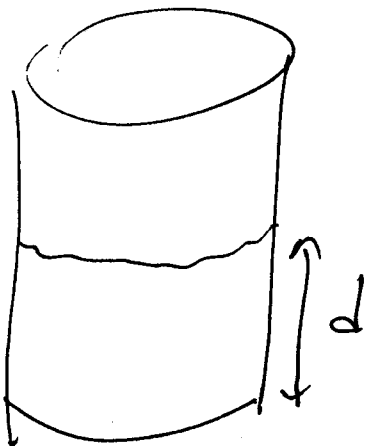
$$\frac{dT}{dt} = k [T(t) - T_{out}(t)]$$

$$T_{out}(t) = 45 + 10 \sin\left(\frac{2\pi t}{24}\right)$$

period = 24 hours

5.2 #17

$$\frac{dV}{dt} = 3 - \frac{k}{5\sqrt{\pi}} \sqrt{V}$$



enter  $3 \frac{ft^3}{sec}$

leaves rate prop  $\sqrt{\text{depth}}$

$$V = \pi 25 d$$

$$\text{leave rate} = k \sqrt{d}$$

# §5.3 Bernoulli's Equation

11/19  
③

$$y' + p(x)y = q(x)y^n$$

$n$  is a real number  $\neq 1$

Trick  $u = y^{1-n}$

will get a linear eq. for  $u$ .

Example  
 $x > 0$   $y' - \frac{1}{2x}y = \frac{1}{2y}$   ~~$\frac{1}{2}y^{-2}$~~

$n = -1$  Try  $u = y^2$

$$\begin{aligned} u' &= 2y y' \\ &= 2y \left( \frac{1}{2x}y + \frac{1}{2y} \right) \end{aligned}$$

$$u' = \frac{1}{x}y^2 + 1$$

$$u' = \frac{1}{x}u + 1$$

$$u' - \frac{1}{x}u = 1$$

$$I.F. = \exp(-\ln|x|) = \frac{1}{|x|} = \frac{1}{x}$$

$$\frac{1}{x} u' - \frac{1}{x^2} u = \frac{1}{x}$$

$u/v$   
④

$$\left(\frac{1}{x} u\right)' = \frac{1}{x}$$

$$\frac{1}{x} u = \int \frac{1}{x} dx = \ln x + C$$

$$u = x \ln x + Cx$$

$$y = \pm \sqrt{u} = \pm \sqrt{x \ln x + Cx}$$

Example

$$\frac{dP}{dt} = kP \left(1 - \frac{P}{b}\right) \quad ||| \quad P(t)$$

$k, b$  constants

$$\frac{dP}{dt} = kP - \frac{k}{b} P^2$$

$$\frac{dP}{dt} - kP = -\frac{k}{b} P^2$$

~~$$u = y^m \quad u' = m y^{m-1} y'$$~~

$$u = P^m \quad u' = m P^{m-1} P'$$

$$u' = m P^{m-1} \left(kP - \frac{k}{b} P^2\right)$$

$$u' = mkp^m - \frac{mk}{b} p^{m+1}$$

2/1/9  
(5)

$$u' = mk u - \frac{mk}{b}$$

$$m+1 > 0$$

$$m = -1$$

$$u' - mk u = -\frac{mk}{b}$$

$$(u e^{-mkt})' = -\frac{mk}{b} e^{-mkt}$$

$$u e^{-mkt} = -\frac{mk}{b} \int e^{-mkt} dt$$

$$= -\frac{mk}{b} \frac{-1}{mk} e^{-mkt} +$$

$$= \frac{1}{b} e^{-mkt} + C$$

$$u e^{kt} = \frac{1}{b} e^{kt} + C$$

$$u = \frac{1}{b} + C e^{-kt}$$

$$u = p^m = p^{-1}$$

$$p(t) = \frac{1}{\frac{1}{b} + C e^{-kt}}$$

# Seasonal Capacity

11/19  
⑥

$$b(t) = 7 + \sin 6t$$

$$\frac{dP}{dt} = kP \left( 1 - \frac{P}{b(t)} \right)$$

Eq.  $P = 0$        $P = b(t) = 7 + \sin 6t$   
not solution

No other eq.

Bernoulli:

$$\frac{dP}{dt} = kP - \frac{k}{b(t)} P^2$$

$$n = 2 \quad 1 - n = -1$$

$$u = P^{-1} \quad u' = -P^{-2} P'$$

$$u' = -P^{-2} \left( kP - \frac{k}{b(t)} P^2 \right)$$

$$u' = -kP^{-1} + \frac{k}{b(t)}$$

$$= -ku + \frac{k}{b(t)}$$

$$\cancel{u'} + ku = \frac{k}{b(t)}$$

$$(u e^{kt})' = e^{kt} \frac{k}{b(t)} \quad \text{U119} \quad \textcircled{7}$$

$$u e^{kt} = \int e^{kt} \frac{k}{7 + \sin 6t} dt$$

Approximate

$$\frac{1}{1+x} \approx 1-x \quad \text{if } x \text{ is small}$$

$$f(x) = \frac{1}{1+x} \quad \text{tangent line at } 0$$

$$f'(x) = \frac{-1}{(1+x)^2}, \quad f'(0) = -1$$

$$f(0) = 1 \quad (0, 1), \quad m = -1$$

$$\text{tangent line} \quad \bullet \quad 1 + (-1)x = 1-x$$

$$\frac{1}{1+x} = 1-x + x^2 - x^3 + x^4 - \dots$$

$$\frac{1}{7 + \sin 6t} = \frac{1}{7 \left(1 + \frac{1}{7} \sin 6t\right)}$$

$$= \frac{1}{7} \frac{1}{1 + \frac{1}{7} \sin 6t}$$

$$\approx \frac{1}{7} (1 - \frac{1}{7} \sin 6t)$$

$\frac{1}{7}$  is small

$$\frac{1}{49}$$


$$\begin{aligned} u e^{kt} &= \int e^{kt} k \frac{1}{7} (1 - \frac{1}{7} \sin 6t) dt \\ &= \frac{k}{7} \int e^{kt} dt \\ &\quad - \frac{k}{49} \int e^{kt} \sin 6t dt \end{aligned}$$

$$u = \frac{1}{7} - \frac{1}{49} - \frac{1}{37} (\sin 6t - 6 \cos 6t) + C e^{-t}$$

$$p = \frac{1}{u}$$