4.3 Continued

~~First~~ Let $N(t)$ = number of cases

Find dif eq for $N(t)$.

	N	$\frac{dN}{dt}$ ← Estimate
1	x	
2	x	
3	x	

25

How does $\frac{dN}{dt}$ depend on N and/or t ?

$$\frac{dN}{dt} = cN$$ ← Assume rate of growth $\propto N$

Plot $\frac{1}{N}\frac{dN}{dt}$ as function of t

One possible model:

$$\frac{1}{N}\frac{dN}{dt} = at + b$$

Use graph to estimate a, b

Another model

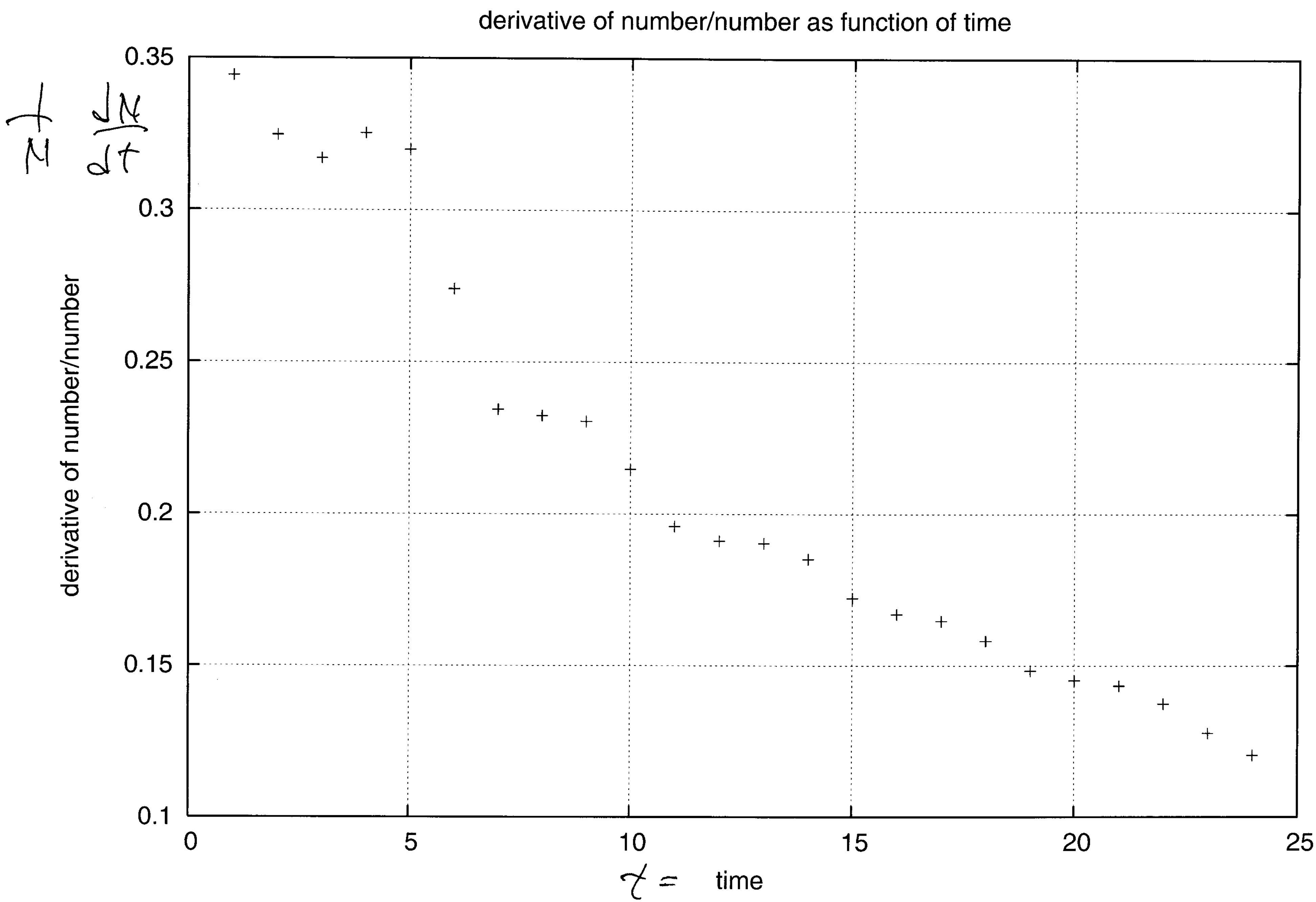
Plot $\frac{dN}{dt}$ as function of N

If $\frac{dN}{dt} = cN^p$, try

log-log plot.

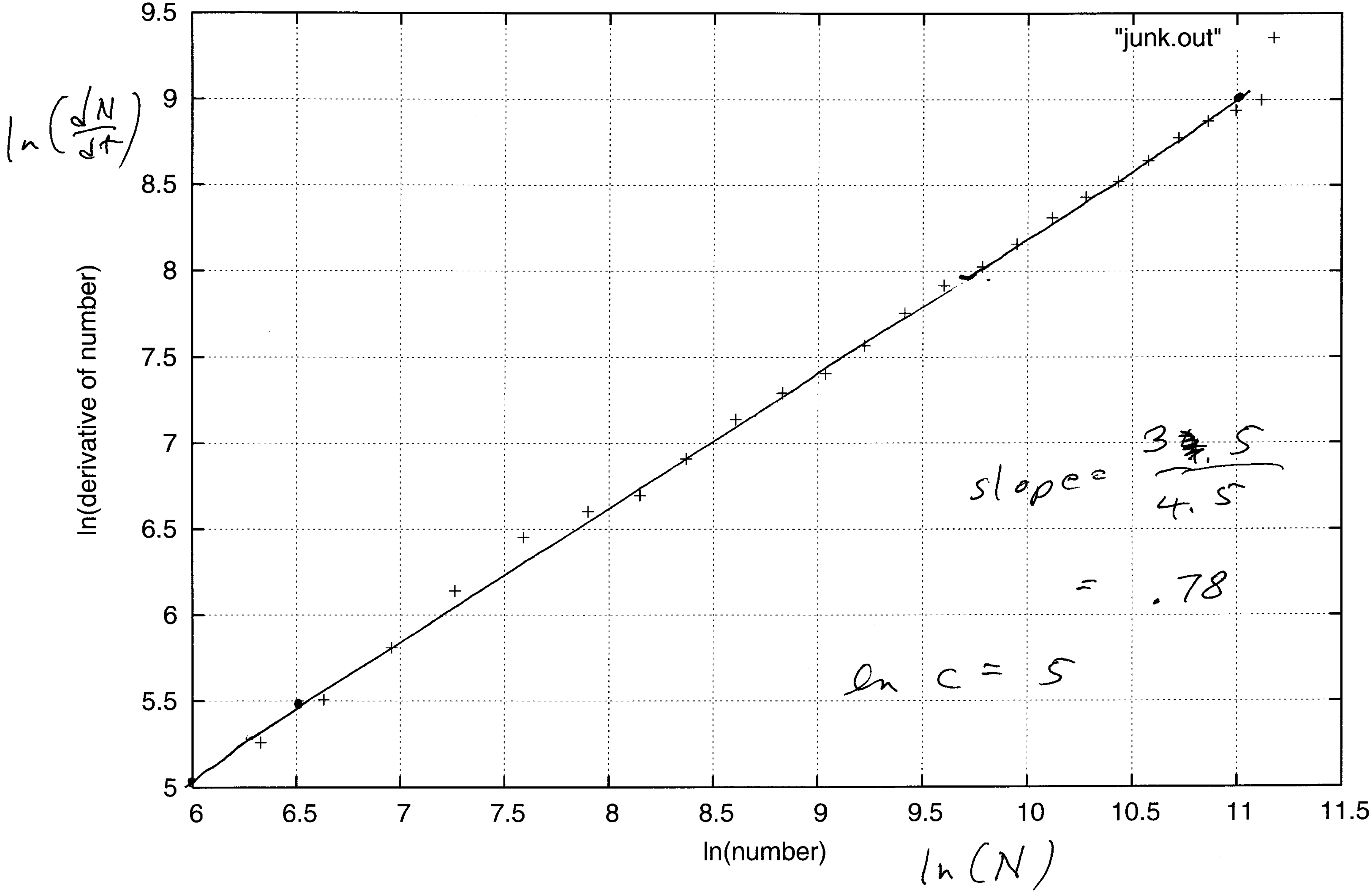
$$\frac{dN}{dt} = c N^{.78}, \quad c = e^5$$

Solve it, compare it to data.



derivative of number/number as function of time
derivative of number/number
time
0.35
0.3
0.25
0.2
0.15
0.1
0
5
10
15
20
25

ln(derivative of number) as function of ln(number)



§ 4.~~[illegible]~~4

$\downarrow +$

Example

Free fall

$$m \frac{dv}{dt} = mg$$

$$\frac{dv}{dt} = g$$

$$v = gt + V_0$$

$$\frac{dx}{dt} = gt + V_0$$

$$x = \frac{1}{2} g t^2 + v_0 t + x_0$$

Example Add friction

Assume: friction $\propto V^2$.

$$m \frac{dv}{dt} = mg - k v^2$$

Equilibria: $V_0 = \sqrt{\frac{mg}{k}}$ terminal velocity

~~m~~ $\frac{dv}{dt} = g - \frac{k}{m} v^2$

$$= g\left(1 - \frac{k}{mg} v^2\right)$$

$$\frac{dv}{dt} = g\left(1 - \frac{v^2}{v_0^2}\right)$$

$$\int \frac{dv}{1 - \frac{v^2}{v_0^2}} = \int g \, dt$$

$$\frac{1}{1 - \frac{v^2}{v_0^2}} = \frac{1}{\left(1 - \frac{v}{v_0}\right)} \frac{1}{\left(1 + \frac{v}{v_0}\right)}$$

$$= \frac{1}{2}\left[\frac{1}{1 - \frac{v}{v_0}} + \frac{1}{1 + \frac{v}{v_0}}\right]$$

...

$$\frac{v}{v_0} = \frac{\exp\left(\frac{2gt}{v_0}\right) - 1}{\exp\left(\frac{2gt}{v_0}\right) + 1}$$

$e^x = \exp(x)$

$$\to 1 \text{ as } t \to \infty$$