8.1, 8.2

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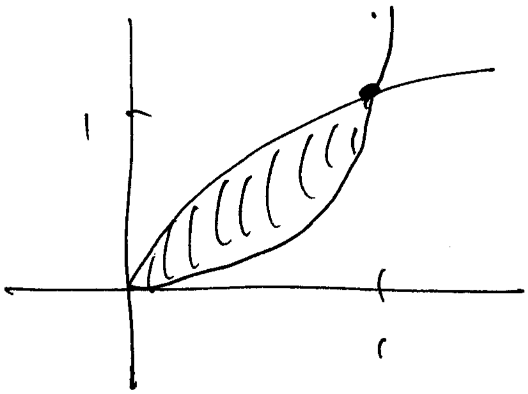
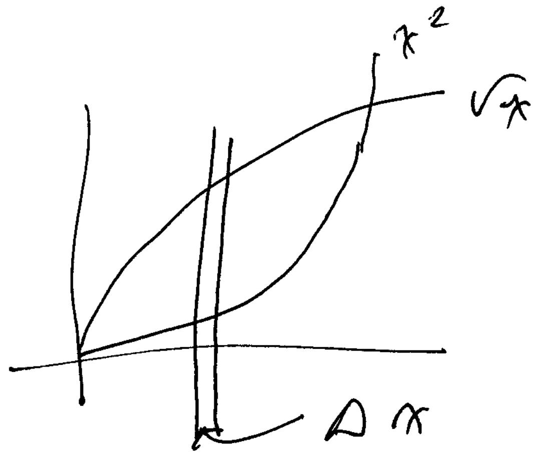
①

① Find area between x^2 and $\sqrt{x}$ for $x \geq 0$





Area of slice $= (\sqrt{x} - x^2)\Delta x$

Total area $= \sum (\sqrt{x} - x^2)\Delta x$

As $\Delta x \to 0$

$$\to \int_0^1 (\sqrt{x} - x^2)\, dx = \dots$$

(2)

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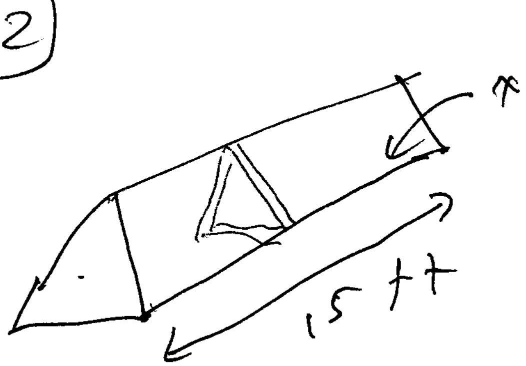
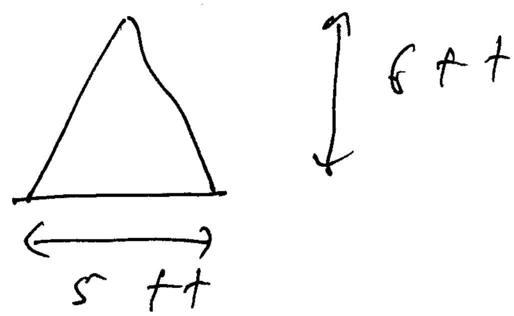
Tent



Cross section



Find its volume.

Slice is triangle
with thickness Δx

$$\text{Slice volume} = (\text{Tri area})\,\Delta x$$

$$= \frac{1}{2}\,5 \cdot 6 \cdot \Delta x$$

$$= 15\,\Delta x$$

$$\text{total vol} \approx \sum 15\,\Delta x$$

$$\to \int_0^{15} 15\,dx = (15)^2$$

3

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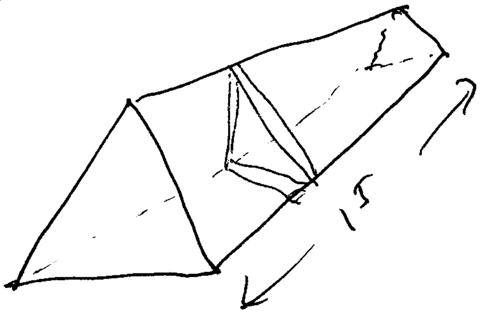
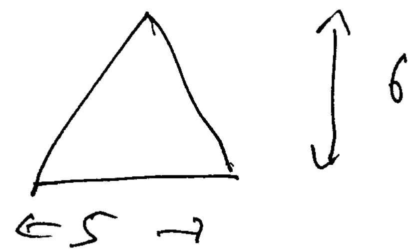
3



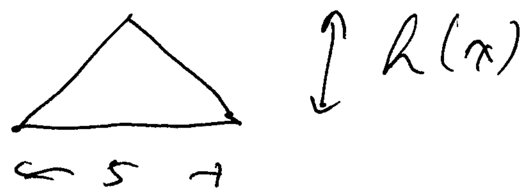
Front



Back

Slice it as before

Slice is



Slice vol $= \frac{1}{2}\, 5\, h(x)\, \Delta x$

total vol $= \sum \cancel{\tfrac{1}{2}}\, \frac{5}{2}\, h(x)\, \Delta x$

$$\to \int_0^{15} \frac{5}{2}\, h(x)\, dx$$

$h(x)$ linear $\quad h(0) = 6, \quad h(15) = 3$

$$h(x) = 6 - \frac{3}{15}x = 6 - \frac{1}{5}x$$

$$\text{Vol} = \int_0^{15} \frac{5}{2}\left(6 - \frac{x}{5}\right) dx$$

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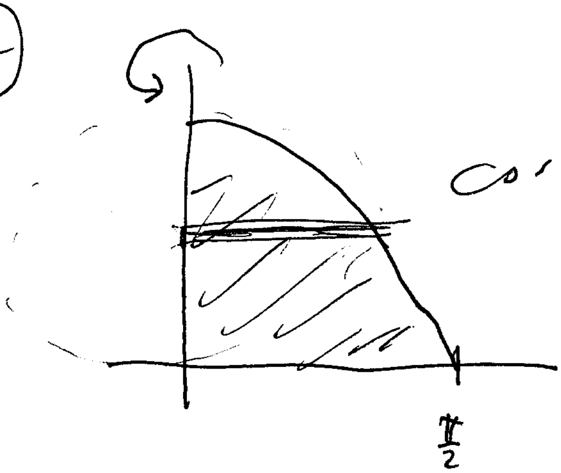
4

4



$0 \le x \le \frac{\pi}{2}$

Rotate this about y axis.

First method Slice perp. to y-axis

slice is a thin disc of radius x and thickness Δy

slice vol $= \pi x^2 \Delta y$

total vol $= \sum \pi x^2 \Delta y$

$$\to \int_0^1 \pi x^2 \, dy$$

$y = \cos x \qquad x = \cos^{-1} y$

$$\text{Vol} = \int_0^1 \pi (\cos^{-1} y)^2 \, dy = 3.5864$$

Second method slice perp. to x-axis

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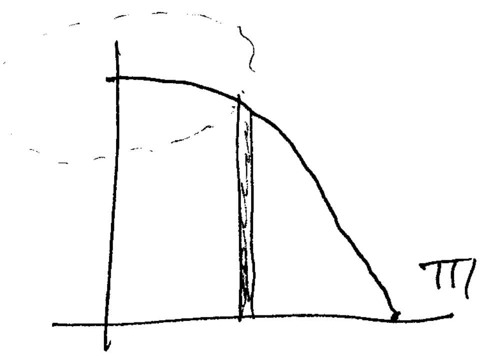
5



Rotate it about y-axis. Get thin ~~x~~ shell.

Unroll the shell and its a ~~the~~ rectangle of thickness Δx

Dim of rectangle: y by $2\pi x$

Slice vol $= 2\pi x\, y\, \Delta x$

total vol $= \sum 2\pi x y\, \Delta x$

$$\to \int_0^{\pi/2} 2\pi x y \, dx$$

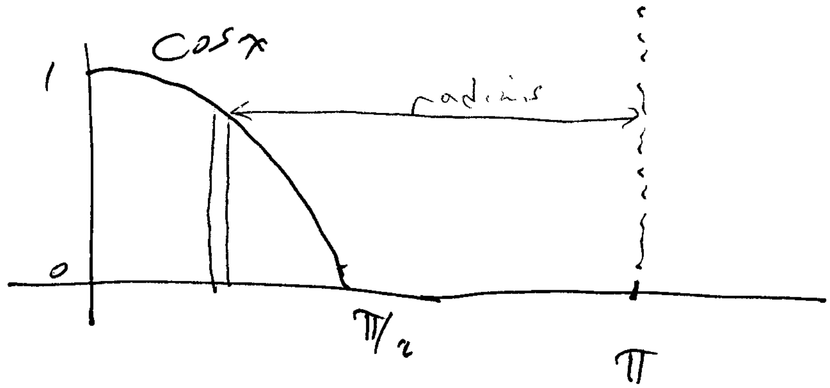
$$= \int_0^{\pi/2} 2\pi x \cos x \, dx$$

$$\int_0^1 \pi (\cos^{-1} y)^2 \, dy \qquad \cancel{x} = \cos^{-1} y$$

$$y = \cos x$$

$$= +\int_0^{\pi/2} \pi \quad x^2 (+\sin x)\, dx \qquad dy = -\sin x \, dx$$

(5)



Rotate about vertical line $x = \pi$

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Slice is a shell.

Thickness Δx

Height $y = \cos x$

Circum. $= 2\pi(\pi - x)$

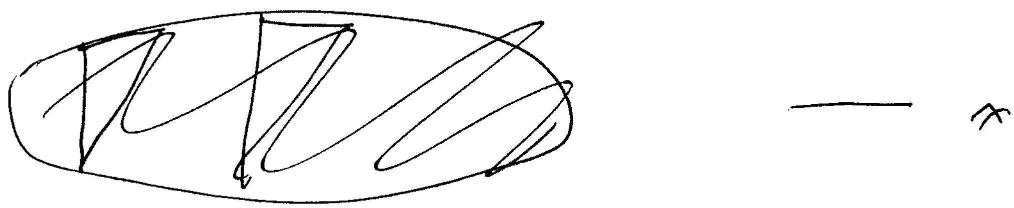
slice vol $= 2\pi(\pi - x)\, y\, \Delta x$

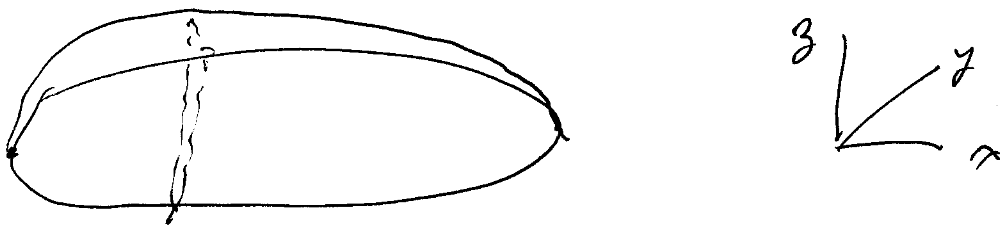
$= 2\pi(\pi - x)\cos x\, \Delta x$

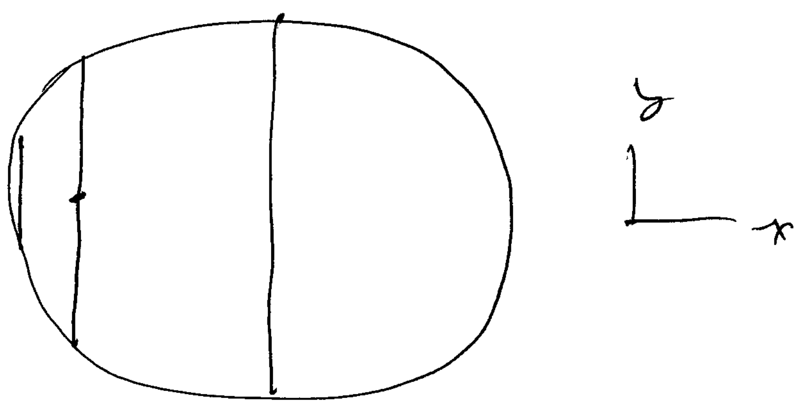
total vol $= \sum 2\pi(\pi - x)\cos x\, \Delta x$

$\to \int_0^{\pi/2} 2\pi(\pi - x)\cos x\, dx$

6 Solid has base which is a circle of radius 2 in x-y plane centered at origin. Cross section perp. to x-axis is an equilateral triangle. Vol = ?







Slice perp. x-axis

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8

Slices are eq. triangles.

Side = 2y

~~slice vol = $\frac{1}{2}(2y)^2 \Delta x$~~

slice vol = $\frac{1}{2}\, 2y\, \frac{2y}{\cancel{2}}\, c$

$= 2y^2 c$

total vol = $\int 2y^2 c\, dx$

$x^2 + y^2 = 4$

$y^2 = 4 - x^2$

vol = $\int_{-2}^{2} 2(4 - x^2)\, c\, dx$

$c = \frac{\sqrt{3}}{2}$