

8.4

Density & mass

8/28
①

Object whose density is not uniform.

Uniform density

$$\text{mass} = \text{volume} \cdot \text{density}$$

Strategy slice so that within a slice the density is constant.

Example Can contains a compressible liquid

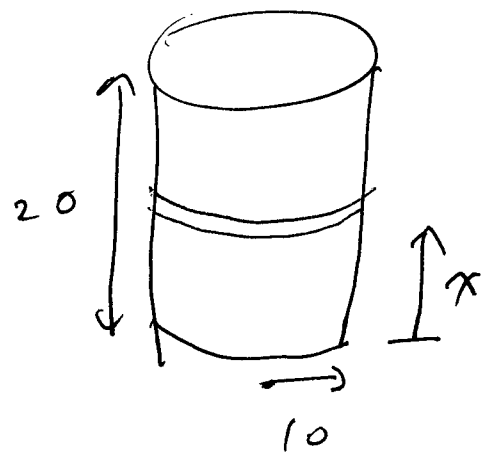
$$\rho(x) = 2 - \frac{x}{25}$$

Find mass of liquid.

$$\text{Slice vol} = \pi 10^2 \Delta x$$

$$\text{Slice mass} = \left(2 - \frac{x}{25}\right) \pi 100 \Delta x$$

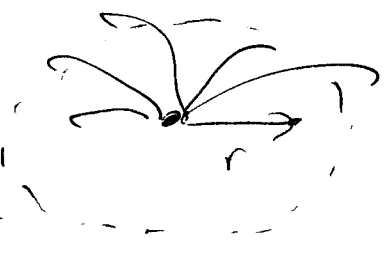
$$\text{total mass} = \int_0^{20} \left(2 - \frac{x}{25}\right) 100 \pi \Delta x$$



Example Broken pipe spraying water over a circle with radius 100 ft.

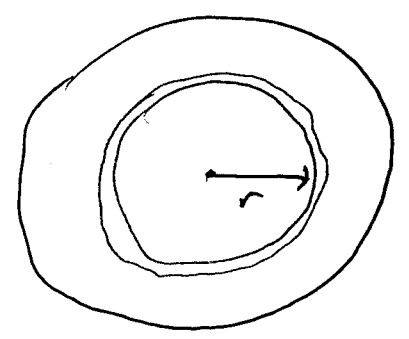
Rate at which water is hitting per unit area is

$$.1 e^{-r/50} \frac{\text{gals}}{\text{ft}^2 \text{ min}}$$

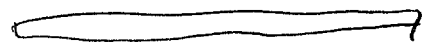


What is the total rate of water loss.

Slice is radius r , thickness Δr .



Slice area = $2\pi r \Delta r$



water flow into slice

$$.1 e^{-r/50} 2\pi r \Delta r$$

$$\left| \begin{array}{l} \pi(r + \Delta r)^2 \\ - \pi r^2 \end{array} \right.$$

total water flow =

$$\int_0^{100} .1 e^{-r/50} 2\pi r \, dr \quad \left| \quad \left(\frac{\text{gal}}{\text{min}} \right) \right.$$