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Exam covers

Calculus 7.5, 7.6

D.f. Eq. 3.3, 3.4

4.1, 4.2, 4.3, 4.4

§ 5.1 Solving Linear D.E.

An equation of the form

$$y' + p(x)\, y = q(x)$$

is called a first order linear dif. eq.

1st order: y'', y''' don't appear

linear: No y^2, $\cos y$, e^y

$$(y')^2, \quad \sqrt{y'}$$

<u>Examples</u>

$$y' + x^2 y = \sin x$$

$$y' = e^x + \frac{y}{x+1}$$

$$(x^2+1)\, y' + \frac{1}{x}\, y = \cos x$$

$$y' + \frac{y}{x(x^2+1)} = \frac{\cos x}{x^2+1}$$

linear

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Non-examples

$$y' + y^2 = e^x$$

$$y y' + x^2 y = \cos x$$

$$y y' + y^2 = \frac{y}{x}$$

Devide by y (careful $y \neq 0$)

$$y' + y = \frac{1}{x} \quad \leftarrow \text{linear}$$

method for solution

Key ~~factor~~ idea:
integrating factor

$$y' + p(x)\, y = g(x)$$

integrating factor $= \exp\left(\int p(x)\, dx\right)$

$= $ function of x

multiply the dif. eq. by I.F.

Example $y' + x y = x$

$p(x) = x \qquad \int p(x)\, dx = \frac{1}{2} x^2$

$$\text{I.F.} = e^{\int p} = e^{\frac{1}{2}x^2}$$

$$\underbrace{y' e^{\frac{1}{2}x^2} + x e^{\frac{1}{2}x^2} y} = x e^{\frac{1}{2}x^2}$$

$$\left(e^{\frac{1}{2}x^2} y\right)' = x e^{\frac{1}{2}x^2}$$

$$\left(e^{\frac{1}{2}x^2} y\right)' = x e^{\frac{1}{2}x^2}$$ 11/7 4

Integrate this

$$e^{\frac{1}{2}x^2} y = \int x e^{\frac{1}{2}x^2} dx$$

$$e^{\frac{1}{2}x^2} y = e^{\frac{1}{2}x^2} + C$$

$$y = 1 + C e^{-\frac{1}{2}x^2}$$

Check

$$y' + xy = x$$

$$y' = -xC e^{-\frac{1}{2}x^2}$$

$$y' + xy = -xCe^{-\frac{1}{2}x^2} + x\left(1 + Ce^{-\frac{1}{2}x^2}\right)$$

$$= x$$ yeah!

Example

$$y' + \frac{1}{x} y - \sin x = 0, \quad x > 0$$

$$y' + \frac{1}{x} y = \sin x$$

$$p(x) = \frac{1}{x} \qquad \int p(x) = \ln x$$

$$e^{\int p} = e^{\ln x} = x$$

multiply dif eq by x

$$\underbrace{x y' + y} = x \sin x$$

$$(x y)' = x \sin x \qquad \boxed{\begin{array}{l} u = x \\ v' = \sin x \end{array}}$$

Integrate

$$x y = \int x \sin x \, dx \qquad \text{parts}$$

...

$$y = -\cos x + \frac{\sin x}{x} + \frac{c}{x}$$

Example

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~~y'~~

$$(1+\sin x)\, y' + 2\cos(x)\, y = 2$$

$$y' + \frac{2\cos x}{1+\sin x}\, y = \frac{2}{1+\sin x}$$

$$p(x) = \frac{2\cos x}{1+\sin x}$$

$u = 1+\sin x$

$du = \cos x\, dx$

$$\int p(x)\,dx = 2\ln(1+\sin x)$$

$$e^{\int p} = e^{2\ln(1+\sin x)}$$

$$= \left[e^{\ln(1+\sin x)}\right]^2$$

$$= (1+\sin x)^2$$

$\times$ integrating factor

$$(1+\sin x)^2\, y' + 2\cos x\,(1+\sin x)\, y = 2(1+\sin x)$$

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$$\left((1+\sin x)^2 y \right)' = 2(1+\sin x)$$

$\int$ it

$$(1+\sin x)^2 y = \int 2(1+\sin x)\,dx$$

$$(1+\sin x)^2 y = 2x - 2\cos x + C$$

$$y = \frac{2x - 2\cos x + C}{(1+\sin x)^2}$$