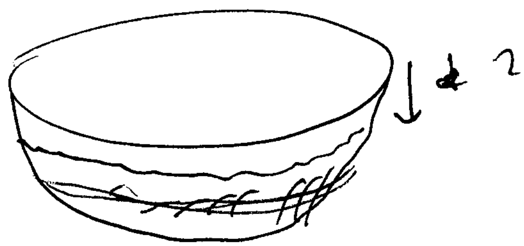
Example

Swimming pool is a hemisphere.



Radius = 20 ft.

~~Fall~~

62.4 $lbs/ft^3$

Water level is ~~3~~ 2 ft below top (ground level)

How much work to pump the water out (to ground level)

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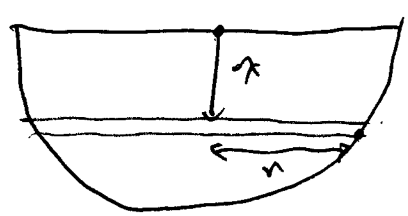
Slice : Disc of radius r

let $x$ be distance below ground level. water: $x = 2$ to $x = 20$



$$x^2 + r^2 = (20)^2$$

Slice vol $= \pi r^2 \Delta x$

Slice weight $= \pi r^2 \Delta x \; 62.4$

Slice work $= \pi \, 62.4 \, r^2 x \, \Delta x$

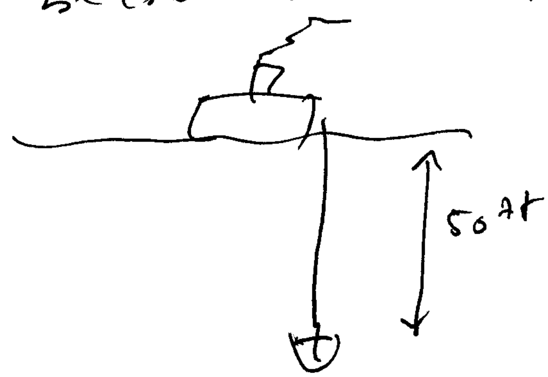
total work $= \displaystyle\int_2^{20} \pi \, 62.4 \, r^2 x \, dx$

$$= \int_2^{20} \pi \, 62.4 \, (400 - x^2) \, x \, dx$$

$$\frac{\text{lbs}}{\text{ft}^3} \quad \text{ft}^2 \quad \text{ft} \quad \text{ft}$$

ft-lb

2 A boat has anchor which weighs 100 lbs. The chain weighs 2 lbs/ft. The chain is 50 ft long and the anchor is directly below the boat.




How much work to raise anchor?

Anchor itself:

50 lbs 100 ft = 5,000 ft-lbs

Chain: Let $x$ be the length of chain already raised.

$x = 0$ to $50$.

Still $50 - x$ ft of chain in water. For $\Delta x$, we need to this piece by distance $\Delta x$.

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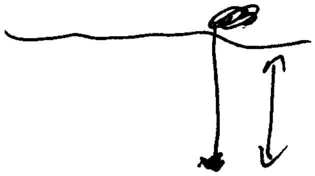
3




height of remaining chain is

$2(50-x)$

Lift it $\Delta x$

slice work $= 2(50-x)\,\Delta x$

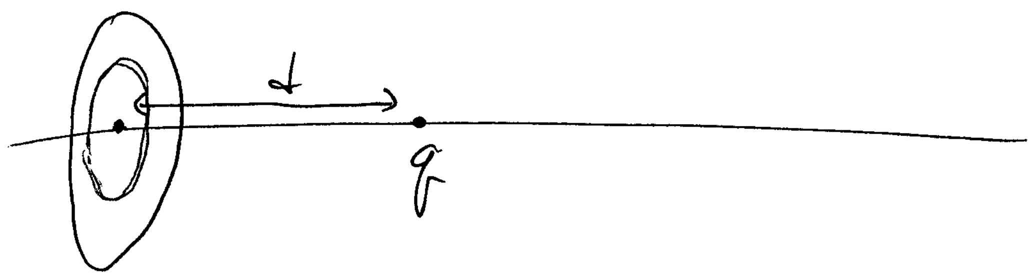
total work $= \displaystyle\int_0^{50} 2(50-x)\,dx$

$= 2 \cdot 50 \cdot 25 = 2{,}500$ ft-lbs

## Example

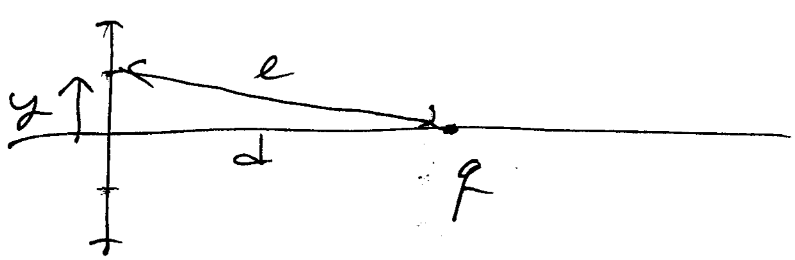
Electrostatic potential between charges $q$ and $Q$ ~~at a~~ seperated a distance $l$ is

$$U = c\,\frac{qQ}{l}$$




Circular plate of radius $R$ has total charge of $Q$. Look at a point along axis of plate at a distance $d$ from plate. Find $U$ there. Charge is uniform on plate.

Slice disc into thin rings.



$$\ell^2 = y^2 + d^2$$

Distance from ring to $q$ is $\ell$

Ring potential is

$$c \frac{q \Delta Q}{\ell}$$

$\Delta Q$ = charge on ring

$$\Delta Q = \frac{\text{area of ring}}{\text{area of plate}} Q$$

$$= \frac{2\pi y \, \Delta y}{\pi R^2} Q$$

Total $U = \int c \frac{q}{\ell} \frac{2\pi y}{\pi R^2} Q \, dy$

$$= \frac{2cqQ}{R^2} \int_0^R \frac{y}{\sqrt{y^2 + d^2}} \, dy$$