

Examples (with parameter) 8/20  
p. 1

①  $f(x) = x^4 + ax^2$   
 $-1 \leq x \leq 1$

Find local min/max, global min/max  
inflection pts.

$$f'(x) = 4x^3 + 2ax$$

Critical pts:  $f'(x) = 0$

$$4x^3 + 2ax = 0$$

$$2x(2x^2 + a) = 0$$

$$x = 0$$

$a \geq 0$  no other solutions

$a < 0$   $x^2 = -\frac{a}{2}$

$$x = \pm \sqrt{-\frac{a}{2}}$$

$$f''(x) = 12x^2 + 2a$$

$$f''(0) = 2a$$

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p. 2

~~also~~  $a > 0$

$$f''(0) > 0$$

So  $x=0$  is local min

$$a < 0$$

$$f''(0) < 0$$

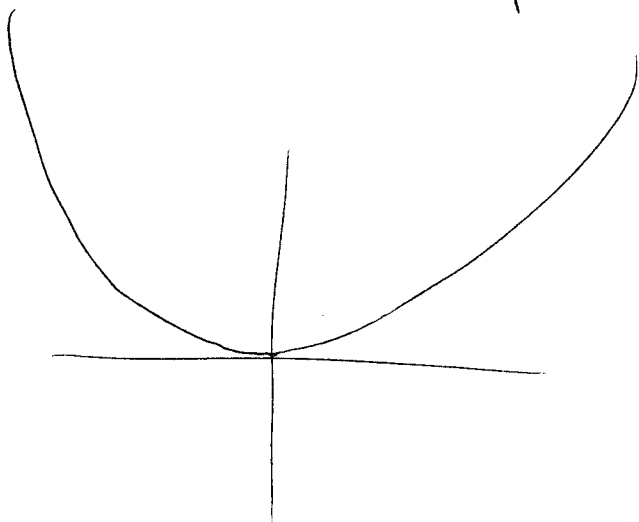
So  $x=0$  is local max.

$$f''\left(\sqrt{-\frac{a}{2}}\right) = 2\left(-\frac{a}{2}\right) + 2a = -4a > 0$$

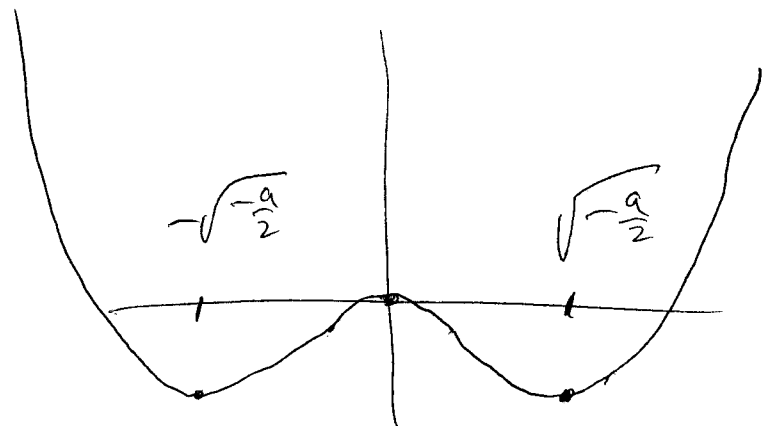
So  $\sqrt{-\frac{a}{2}}$  is a local min.

Likewise  $-\sqrt{-\frac{a}{2}}$  is a local min.

qualitative change at  $a=0$



$a > 0$



$a < 0$