

Global min :

$a > 0$ at $x = 0$, $f(0) = 0$

$a < 0$ Compare

$$x = \pm 1, \quad x = \pm \sqrt{\frac{-a}{2}}$$

If $a < -2$, there are no ~~only~~ local mins in $(-1, 1)$ with $f'(x) = 0$.

$a < -2 \Rightarrow \pm 1$ are global mins

$$f(\pm 1) = \cancel{4 + 2a} + 1 + a$$

$a \geq -2$, $a < 0$

Compare

endpts

$$x = \pm 1$$

$$x = \pm \sqrt{\frac{-a}{2}}$$

$$f(\pm 1) = 1 + a \quad f(\pm \sqrt{\frac{-a}{2}})$$

$$= \frac{a^2}{4} + a\left(\frac{-a}{2}\right) = -\frac{a^2}{4}$$

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$$\int_0^1 (x^2 - \pi) dx$$

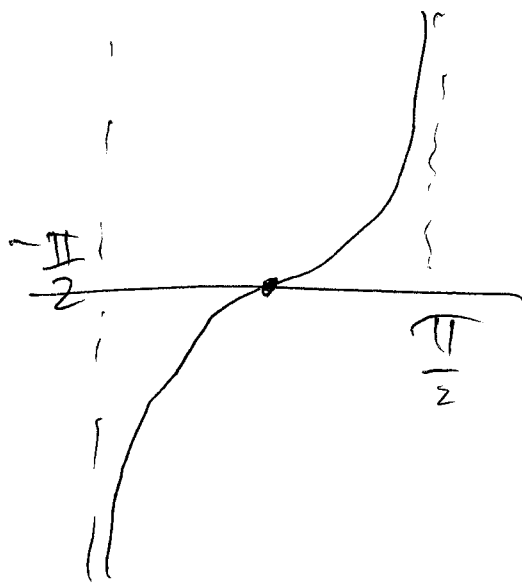
$$= \left[\frac{1}{3} x^3 - \pi x \right]_0^1$$

$$= \frac{1}{3} 1^3 - \pi \cdot 1 = \frac{1}{3} - \pi$$

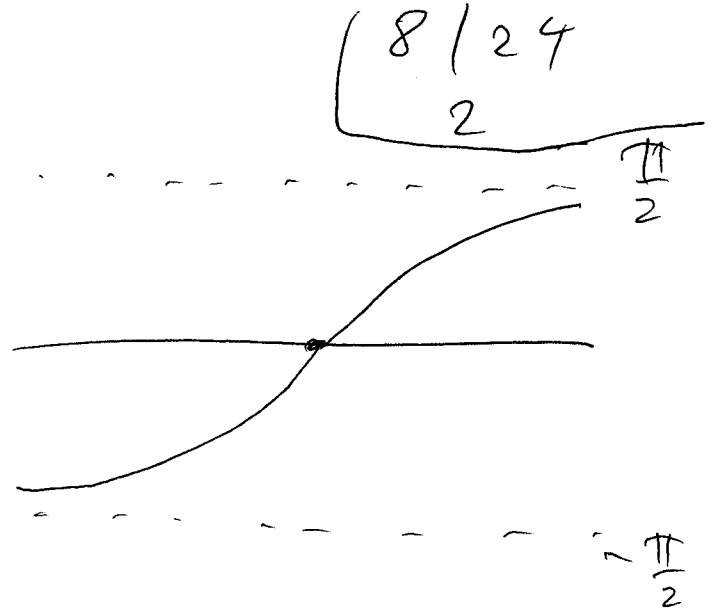
$$\int_0^1 \frac{1}{1+x^2} dx = \tan^{-1} x \Big|_0^1$$

$$= \tan^{-1}(1) - \tan^{-1}(0)$$

$$= \frac{\pi}{4} - 0 = \frac{\pi}{4}$$



$\tan x$



$\tan^{-1} x$

$$\frac{d}{dx} \int_a^x f(x) dx$$

↑
dummy variable

$$\textcircled{1} \quad \frac{d}{dx} \int_0^x \sin u \, du$$

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Brute force :

$$= \frac{d}{dx} \left[-\cos u \right]_{u=0}^{u=x}$$

$$= \frac{d}{dx} \left[-\cos x + \cos 0 \right]$$

$$= \sin x$$

Better :

$$\sin x$$

$$\textcircled{2} \quad \frac{d}{dx} \int_2^x \sin(3t^2) \, dt$$

$$= \sin(3x^2)$$