(1) $\int x \sin(x^2)\, dx$

$$\frac{d}{dx} \cos(x^2) = -\sin(x^2)\; 2x$$
$$= -2x \sin(x^2)$$

$$\int x \sin(x^2)\, dx = -\tfrac{1}{2} \cos(x^2) + C$$

Formal sub:

$u = x^2$

$u' = 2x$, $du = 2x\, dx$

$$\int \sin(x^2)\, x\, dx = \int \sin(u)\, \tfrac{1}{2}\, du$$

$$= \tfrac{1}{2}(-\cos u) = -\tfrac{1}{2} \cos(x^2) + C$$

8/27
p. 2

(2) $\int \sec^2 (x^{1/2}) \underbrace{x^{-1/2}\, dx}$

$u = x^{1/2}$, $du = \frac{1}{2} \underbrace{x^{-1/2}\, dx}$

$= \int \sec^2(u)\, 2\, du$

$= 2 \tan(u) + C$

$= 2 \tan(x^{1/2}) + C$

(3) $\int \frac{\cos t}{1+\sin^2 t}\, dt$

$\boxed{u = \cancel{1+\sin^2 t}}$

$u = \sin t$ $\quad du = \cos t\, dt$

$= \int \frac{du}{1+u^2} = \tan^{-1} u + C$

$= \tan^{-1}(\sin t) + C$

8/27
p. 3

④ $$\int \frac{x+3}{x^2+6x+2}\,dx$$

$$u = x^2+6x+2$$
$$u' = 2x+6 \qquad du = (2x+6)\,dx$$

$$= \int \frac{\frac{1}{2}\,du}{u} = \frac{1}{2}\ln|u| + C$$
$$= \frac{1}{2}\ln|x^2+6x+2| + C$$

$$\int \frac{dx}{x} = \ln|x|$$

⑤ $$\int \frac{x}{x^2+6x+2}\,dx$$

$$= \int \frac{x+3}{x^2+6x+2}\,dx - \int \frac{3}{x^2+6x+2}\,dx$$

$$= \int \frac{x+3}{x^2+6x+2}\,dx - 3\int \frac{dx}{x^2+6x+2}$$

8/27 p.4

$$\int \frac{dx}{x^2+6x+2}$$

$b^2 - 4ac =$
$36 - 8$
> 0

= partial fractions (later)

<u>Non-obvious subs</u>

$$\int \frac{dx}{1+x^{1/3}}$$

$u = 1 + x^{1/3}$

$u' = 1/3 \; x^{-2/3}$

$du = \frac{1}{3} x^{-2/3} \, dx \Rightarrow dx = 3x^{2/3} \, du$

$$\int \frac{3x^{2/3} \, du}{u}$$

$u - 1 = x^{1/3}$

$(u-1)^3 = x$

$$= \int \frac{3(u-1)^2 \, du}{u}$$

$$= 3 \int \frac{u^2 - 2u + 1}{u} \, du = 3 \int \left(u - 2 + \frac{1}{u}\right) du$$

$$= 3\left[\frac{1}{2}\left(1+x^{1/3}\right)^2 - 2\left(1+x^{1/3}\right) + \ln\left|1+x^{1/3}\right|\right] + C$$

Definite integrals

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

$F' = f$ "x - values"

① $\int_0^2 x e^{x^2}\,dx$

$$\int x e^{x^2}\,dx = \frac{1}{2} e^{x^2} + C$$

$$\frac{1}{2} e^{x^2} \Big|_{x=0}^{x=2} = \frac{1}{2} e^{2^2} - \frac{1}{2} e^0$$

$$= \frac{1}{2} e^4 - \frac{1}{2}$$

<u>2nd method</u>

$u = x^2$ $\quad du = 2x\,dx$

$$\int_0^2 x e^{+x^2}\,dx = \int_0^4 e^{+u} \frac{1}{2}\,du$$

$$= \frac{1}{2} e^u \Big|_{u=0}^{u=4} = \frac{1}{2} e^4 - \frac{1}{2} e^0$$

Sophisticated Subs

8/27 p. 6

Let $$g(a) = \int_{-a}^{a} \frac{e^{x/a}}{x^2}\, dx$$

How does $g(a)$ depend on a?

$u = \frac{x}{a}$, $\quad du = \frac{1}{a}\, dx$

$$g(a) = \int_{-1}^{+1} \frac{e^u \, a\, du}{x^2} \qquad \Bigg| \quad x = ua$$

$$= \int_{-1}^{1} \frac{e^u \, a\, du}{u^2 a^2}$$

$$= \frac{1}{a} \int_{-1}^{1} \frac{e^u\, du}{u^2} = \frac{c}{a}$$

$$c = \int_{-1}^{1} \frac{e^u\, du}{u^2}$$

8/27
p. 7

Given that

$$\int_{-\infty}^{\infty} e^{-x^2}\,dx = \sqrt{\pi}$$

Find

$$\int_{-\infty}^{\infty} e^{-cx^2}\,dx$$

$$u^2 = c x^2 \qquad \boxed{u = \sqrt{c}\,x}$$

$$u' = \sqrt{c} \qquad du = \sqrt{c}\,dx$$

$$\int_{-\infty}^{\infty} e^{-u^2} \frac{1}{\sqrt{c}}\,du$$

$$= \frac{1}{\sqrt{c}} \int_{-\infty}^{\infty} e^{-u^2}\,du = \frac{\sqrt{\pi}}{\sqrt{c}}$$