

Suppose $\int_0^1 f(x) dx = 2$ 8/29 p1

Find $\int_0^L f\left(\frac{x}{L}\right) dx$

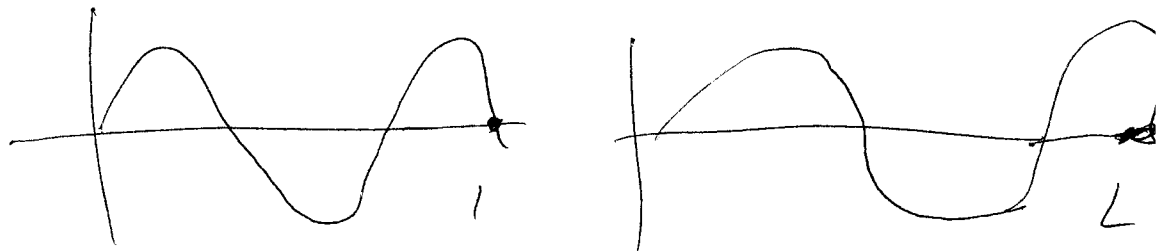
$$u = \frac{x}{L}$$

$$du = \frac{dx}{L}$$

$\int_0^1 f(u) L du = L \int_0^1 f(u) du$

$$= 2L$$

$$\frac{L > 1}{f\left(\frac{x}{L}\right)}$$



$$\int_0^1 \frac{x \, dx}{x^2 + 1}$$

$$u = x^2$$

$$du = 2x \, dx$$

$$\Rightarrow \int_0^1 \frac{\frac{1}{2} du}{u^2 + 1}$$

$$= \frac{1}{2} \int_0^1 \frac{du}{u^2 + 1}$$

$$= \frac{1}{2} \tan^{-1} u \Big|_0^1 = \frac{1}{2} \left(\frac{\pi}{4} - 0 \right)$$

$$= \frac{\pi}{8}$$

$$\int \frac{e^x}{\sqrt{1-e^x}} \, dx$$

$$u = 1 - e^x$$

$$du = -e^x \, dx$$

$$\Rightarrow \int \frac{-du}{\sqrt{u}}$$

$$= - \int u^{-1/2} \, du$$

$$= -2 u^{1/2} + C = -2\sqrt{1-e^x} + C$$

$$\int \frac{e^{x/2} dx}{\sqrt{1-e^x}}$$

$$\begin{array}{l} \text{8(29) p 3} \\ u = e^{\frac{x}{2}} \end{array}$$

$$du = \frac{1}{2} e^{\frac{x}{2}} dx$$

$$\int = \int \frac{2 du}{\sqrt{1-u^2}}$$

$$\begin{array}{l} e^x = (e^{\frac{x}{2}})^2 \\ = u^2 \end{array}$$

$$= 2 \sin^{-1}(u) = 2 \sin^{-1}(e^{\frac{x}{2}}) + C$$

7.2 Integration by parts (8/29 p4)

$$(uv)' = u'v + uv'$$

$$uv = \int u'v \, dx + \int uv' \, dx$$

$$\int x e^{2x} \, dx$$

$$u = x \quad v' = e^{2x}$$

$$u' = 1 \quad v = \frac{1}{2} e^{2x}$$

$$= \frac{1}{2} x e^{2x} - \int \frac{1}{2} e^{2x} \, dx$$

$$= \frac{1}{2} x e^{2x} - \frac{1}{4} e^{2x} + C$$

$$\int x^2 e^{2x} \, dx$$

$$u = x^2 \quad v' = e^{2x}$$

$$u' = 2x \quad v = \frac{1}{2} e^{2x}$$

$$= \frac{1}{2} x^2 e^{2x} - \int 2x \cdot \frac{1}{2} e^{2x} \, dx$$

$$= \frac{1}{2} x^2 e^{2x} - \left[\frac{1}{2} x e^{2x} - \frac{1}{4} e^{2x} \right] + C$$

$$= \frac{1}{2} x^2 e^{2x} - \frac{1}{2} x e^{2x} + \frac{1}{4} e^{2x} + C$$

Feynmann's trick

Integration by differentiation

~~Step~~ $\int e^{ax} dx = \frac{1}{a} e^{ax}$

d
da this!

$$\int x e^{ax} dx = -\frac{1}{a^2} e^{ax} + \frac{x}{a} e^{ax}$$

Set $a = 2$

$$\int x e^{2x} dx = -\frac{1}{4} e^{2x} + \frac{x}{2} e^{2x}$$

$$\int \ln x \, dx = \int 1 \cdot \ln x \, dx$$

$$\left(\begin{array}{l} u = \ln x, \quad v' = 1 \\ u' = \frac{1}{x}, \quad v = x \end{array} \right.$$

$$x \ln x - \int \frac{1}{x} x \, dx$$

$$= x \ln x - x + C$$

$$\int x \ln x \, dx$$

$$\left(\begin{array}{l} u = \ln x \quad v' = x \\ u' = \frac{1}{x} \quad v = \frac{1}{2} x^2 \end{array} \right.$$

$$\int \frac{1}{x} \cdot \frac{1}{2} x^2 \, dx$$

$$= \frac{1}{2} x^2 \ln x - \frac{1}{2} \int x \, dx$$

$$= \frac{1}{2} x^2 \ln x - \frac{1}{4} x^2 + C$$

Circular

8/29 p7

$$\int e^{2x} \sin(3x) dx$$

$$u = \sin(3x) \quad v' = e^{2x}$$

$$u' = 3 \cos(3x) \quad v = \frac{1}{2} e^{2x}$$

$$= \frac{1}{2} \sin(3x) e^{2x} - \frac{3}{2} \int \cos(3x) e^{2x} dx$$

$$u = \cos(3x) \quad v' = e^{2x}$$

$$u' = -3 \sin(3x) \quad v = \frac{1}{2} e^{2x}$$

$$= \frac{1}{2} \sin(3x) e^{2x} - \frac{3}{2} \left[\frac{1}{2} \cos(3x) e^{2x} - \int (-3) \sin(3x) \frac{1}{2} e^{2x} dx \right]$$

~~Let~~

$$= \frac{1}{2} \sin(3x) e^{2x} - \frac{3}{4} \cos(3x) e^{2x}$$

$$- \frac{9}{4} \int \sin(3x) e^{2x} dx$$

$$\text{Let } I = \int e^{2x} \sin(3x) dx$$

$$I = \frac{1}{2} \sin(3x) e^{2x} - \frac{3}{4} \cos(3x) e^{2x} - \frac{9}{4} I$$

$$\frac{13}{4} I = \dots$$

$$I = \frac{4}{13} \left[\frac{1}{2} \sin(3x) e^{2x} - \frac{3}{4} \cos 3x e^{2x} \right] + C$$

Use $\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$

to find $\int_{-\infty}^{\infty} x^2 e^{-x^2} dx$

$$u = e^{-x^2}$$

$$v' = x^2$$

$$u' = -2x e^{-x^2}$$

$$v = \frac{1}{3} x^3$$

Bad idea

$$u = x$$

$$u' = 1$$

$$v' = x e^{-x^2}$$

$$v = -\frac{1}{2} e^{-x^2}$$

8/29
p 9

$$= -\frac{1}{2} x e^{-x^2} \Big|_{-\infty}^{\infty} - \int_{-\infty}^{\infty} \left(-\frac{1}{2}\right) e^{-x^2} dx$$

$$= 0 - 0 + \frac{1}{2} \sqrt{\pi}$$

$$\int e^{-ax^2} dx = \frac{\sqrt{\pi}}{\sqrt{a}}$$

use Feynman trick