Example $\theta(t)$

$$\frac{d\theta}{dt} = \sin\theta$$

when is $\theta(t) = c$ a solution ?

when $\sin c = 0$

$c = \dots -2\pi, -\pi, 0, \pi, 2\pi \dots$

A constant solution is called an equilibrium solution.

Such such a solution is stable if for nearby initial conditions the solution approaches the eq. solution, as $t \to +\infty$.

unstable if nearby solutions move away from it.

$$\frac{dy}{dx} = g(y)$$

$y = c$ is eq. solution if and if $g(c) = 0$.

Example

$$\frac{dy}{dt} = c(a-y)y$$

$$\frac{dy}{dt} = -2(3-y)y$$

3 is eq. sol that is unstable

0 is stable

$$\frac{dy}{dt} = 2(3-y)y$$

$$\frac{dy}{dt} = -2(3-y)y$$

$$\frac{dy}{(3-y)y} = -2\,dt$$

$$\int \frac{dy}{(3-y)\,y} = -2t + C$$

Partial fractions : algebra

$$\frac{1}{(3-y)\,y} = \frac{A}{3-y} + \frac{B}{y}$$

Find A, B

$$\frac{1}{(3-y)\,y} = \frac{Ay + B(3-y)}{(3-y)\,y}$$

$$1 = Ay + B(3-y)$$

$$1 = Ay - By + 3B$$

$$\Rightarrow \underbrace{1 = 3B}_{\text{const}}, \quad \underbrace{0 = A - B}_{y\text{-eq.}}$$

9/14

4

$1 = 3B, \qquad A - B = 0$

$B = 1/3, \qquad A = 1/3$

$$\frac{1}{(3-y)\,y} = \frac{\frac{1}{3}}{3-y} + \frac{\frac{1}{3}}{y}$$

$$\int \frac{dy}{(3-y)\,y} = \frac{1}{3}\int \frac{dy}{3-y} + \frac{1}{3}\int \frac{dy}{y}$$

$$= -\frac{1}{3}\ln|3-y| + \frac{1}{3}\ln|y|$$

So $-\frac{1}{3}\ln|3-y| + \frac{1}{3}\ln|y| = -2t + C$

$$-\ln|3-y| + \ln|y| = -6t + C'$$

$$e^{-\ln|3-y| + \ln|y|} = e^{C'} e^{-6t}$$

$$\frac{|y|}{|3-y|} = D e^{-6t}, \quad D > 0$$

Consider <u>cases</u>.