

Exam wed

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①

T. 1, T. 2 calc

L. 1, L. 2, L. 3 dit eq.

2nd fundamental thm

No "functions with parameters"

No complex numbers

L. 3 "due" wed

2.1 due Fri

Office Hours: Tues 4:30-5:00

$$\frac{|y|}{|3-y|} = D e^{-6t}$$

$$D > 0$$

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Case $y > 3$

$$|-2| = -(-2) = 2$$

$$|y| = y, \quad |3-y| = -(3-y) = y-3$$

$$\frac{y}{y-3} = D e^{-6t}$$

$$y = D e^{-6t} \quad y - 3 D e^{-6t}$$

$$(1 - D e^{-6t}) y = -3 D e^{-6t}$$

$$y = \frac{-3 D e^{-6t}}{1 - D e^{-6t}}$$

$$\text{As } t \rightarrow \infty, \quad e^{-6t} \rightarrow 0$$

$$y \rightarrow \frac{0}{1-0} = 0$$

§ 7.4 Partial Fractions

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Rational functions

$$\frac{P(x)}{Q(x)}$$

P, Q polynomials

If degree of P is \geq
degree of Q , do long division.

Example

$$\frac{x^3 + 2x - 3}{x + 2}$$

$$\begin{array}{r} x^2 - 2x + 6 \\ x + 2 \overline{) \begin{array}{r} -x^3 + 0x^2 + 2x - 3 \\ x^3 + 2x^2 \\ \hline -2x^2 + 2x - 3 \\ -2x^2 - 4x \\ \hline 6x - 3 \\ 6x + 12 \\ \hline -15 \end{array}} \end{array}$$

$$\frac{x^3 + 2x - 3}{x + 2} = x^2 - 2x + 6 - \frac{15}{x + 2}$$

Algebra facts

① $Q(x)$ can be factored into a product of linear terms $(x-c)$ and irreducible quadratic terms.

② Quadratic polynomial:

one root: $x^2 + bx + c = (x-\alpha)^2$

two roots: $x^2 + bx + c = (x-\alpha)(x-\beta)$

no real roots: (irreducible)

$$x^2 + bx + c = (x-\alpha)^2 + \beta^2$$

(complete the square)

$$\frac{P(x)}{Q(x)} = \text{sum of terms}$$

Examples what to include

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$$\textcircled{1} \frac{8x^2 + 3x - 2}{x^2(x^2 + 2)(x - 5)}$$

$$= \frac{A_1}{x} + \frac{A_2}{x^2} + \frac{Ax + B}{x^2 + 2} + \frac{C}{x - 5}$$

$$\textcircled{2} \frac{3x^2 - 2x}{(x - 2)(x + 2)(x^2 + x + 10)^2}$$

$$= \frac{A}{x - 2} + \frac{B}{x + 2} + \frac{Cx + D}{x^2 + x + 10}$$

$$+ \frac{Ex + F}{(x^2 + x + 10)^2}$$

~~$\frac{A_1}{(x - c)^2} + \frac{A_2}{(x - c)^2}$~~

$$\textcircled{1} \int \frac{2x+1}{(x-1)(x+2)} dx$$

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$$= \int \left(\frac{A}{x-1} + \frac{B}{x+2} \right) dx$$

Algebra

$$\frac{2x+1}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2}$$

$$= \frac{A(x+2) + B(x-1)}{(x-1)(x+2)}$$

$$\text{So } 2x+1 = A(x+2) + B(x-1)$$

$$\underline{2x+1} = \underline{Ax} + 2A + \underline{Bx} - B$$

$$1 = 2A - B \quad (\text{const})$$

$$2 = A + B \quad (x)$$

$$\text{add them: } 3 = 3A \quad A = 1$$

$$B = 1$$

$$\int \frac{2x+1}{(x-1)(x+2)} dx = \int \left(\frac{1}{x-1} + \frac{1}{x+2} \right) dx$$

$$= \ln|x-1| + \ln|x+2| + C \quad \left. \begin{array}{l} 9/17 \\ \textcircled{7} \end{array} \right\}$$

$$\textcircled{2} \int \frac{x^2+5}{x(x^2+1)} dx$$

$$\frac{x^2+5}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$$

$$= \frac{A(x^2+1) + (Bx+C)x}{x(x^2+1)}$$

$$x^2+5 = A(x^2+1) + (Bx+C)x$$

$$x^2+5 = Ax^2 + A + Bx^2 + Cx$$

$$\text{const:} \quad 5 = A$$

$$x: \quad 0 = C$$

$$x^2: \quad 1 = A + B$$

$$B = -4$$

$$\int \frac{x^2+5}{x(x^2+1)} dx = \int \left(\frac{5}{x} + \frac{-4x}{x^2+1} \right) dx$$

$$= 5 \ln|x| - 2 \ln|x^2+1| + C$$