$$\int \frac{1}{x-c}\,dx = \ln|x-c|$$

$$\int \frac{1}{(x-c)^2}\,dx = \frac{-1}{x-c}$$

$$\int \frac{Ax+B}{q(x)}\,dx \qquad q(x) \text{ irreducible}$$

Complete the square.

Use sub to turn it into

$$\int \frac{du}{1+u^2} = \tan^{-1} u$$

Example

$$\int \frac{(x-1)}{x^2+2x+5}\,dx$$

$$= \int \frac{x-1}{(x+1)^2+4}\,dx$$

$$\int \frac{du}{4u^2+4} = \frac{1}{4}\tan^{-1} u$$

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Sub $(x+1)^2 + 4 = 4u^2 + 4$

$x+1 = 2u \qquad dx = 2\,du$

~~$du = \frac{1}{2}\,d$~~

$$\int = \int \frac{2u-2}{4u^2+4}\, 2\,du$$

$$= \frac{2}{4}\, 2 \int \frac{u-1}{u^2+1}\, du$$

$$= \int \frac{u}{u^2+1}\, du - \int \frac{du}{u^2+1}$$

$$= \frac{1}{2} \ln(u^2+1) - \tan^{-1} u + C$$

$$= \frac{1}{2} \ln\left(\frac{1}{4}(x+1)^2 + 1\right) - \tan^{-1}\left(\frac{x+1}{2}\right) + C$$

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$$\int \frac{dx}{\sqrt{-x^2+6x-5}}$$

$$= \int \frac{dx}{\sqrt{-(x-3)^2+4}}$$

$$\int \frac{du}{\sqrt{1-u^2}}$$

$$\int \frac{du}{\sqrt{4-4u^2}}$$

$$(x-3)^2 = 4u^2$$

$$x-3 = 2u$$

$$dx = 2\,du$$

$$= \int \frac{2\,du}{\sqrt{4-4u^2}} = \int \frac{du}{\sqrt{1-u^2}}$$

$$= \sin^{-1}(u) + C = \sin^{-1}\left(\frac{x-3}{2}\right) + C$$

$$-x^2+6x-5 = -(x^2-6x+5)$$

$$= -((x-3)^2 - 4)$$

$$= -(x-3)^2 + 4$$

2.2 cont

Pop of ~~T~~ Tucson $P(t)$

satisfies

$$P'(t) = k\,P(t)$$

General sol $P(t) = C\,e^{kt}$

2000 720,000

2005 780,000

$t=0$ is year 2000

$$P(0) = 720,000$$

$$P(0) = C\,e^{0} = C$$

$$C = 720,000$$

$$P(5) = 780,000$$

$$C\,e^{5k} = 780,000$$

$$e^{5k} = \frac{780,000}{720,000} = \frac{78}{72}$$

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$$5k = \ln \frac{78}{72}$$

$$k = \frac{\ln \frac{78}{72}}{5}$$

Doubling

$$P(t_D) = 2\,P(0)$$

$$C e^{k t_D} = 2C\,e^{0}$$

$$e^{k t_D} = 2$$

$$k t_D = \ln 2$$

$$t_D = \frac{\ln 2}{k}$$