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1

Example that wouldn't die

$$\frac{dy}{dt} = -2y(3-y)$$

$$\frac{|y|}{|3-y|} = D e^{-6t}$$

$y > 3$

$$\frac{y}{y-3} = D e^{-6t}$$

I.C. $y(0) = 4$

$$\frac{4}{4-3} = D e^{0}$$

$$D = 4$$

$$\frac{y}{y-3} = 4 e^{-6t}$$

$$y = 4e^{-6t}y - \cancel{3}12e^{-6t}$$

$$y - 4e^{-6t}y = -3e^{-6t}$$

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2

$$y = \frac{-12e^{-6t}}{1-4e^{-6t}}$$ ☆

$t=0$ $\quad \frac{-12}{1-4} = \frac{-12}{-3} = 4$

$t = .1$

At some t, $\quad 1 - 4e^{-6t} = 0$

$$e^{-6t} = \frac{1}{4}$$

$$e^{6t} = 4$$

$$6t = \ln 4$$

$$t = \frac{\ln 4}{6} \quad \text{blow up!}$$

☆ valid only for

$$t < \frac{\ln 4}{6}$$

§ 2.3 Logistic Equation

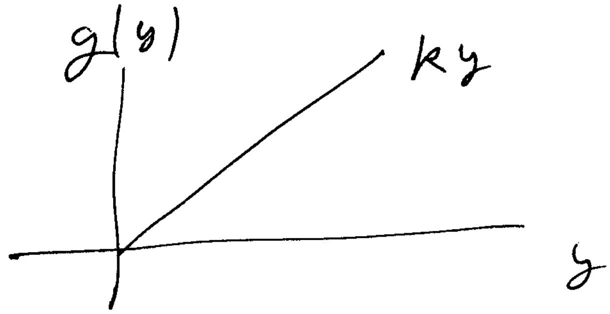
Population growth

$y(t)$ = population

simplest model (2.2)

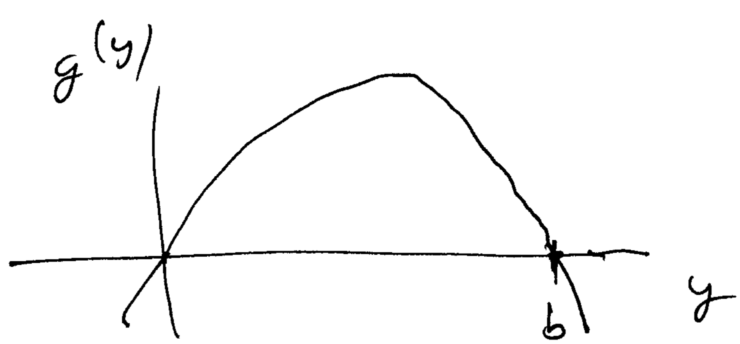
$$\frac{dy}{dt} = ky, \quad k > 0$$

$$y(t) = y(0)\, e^{kt}$$



$$\frac{dy}{dt} = g(y)$$

Suppose the environment can only sustain a population of $y = b$



$$a\,y\,(b - y)$$

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$$\frac{dy}{dt} = a y (b-y)$$

$b > 0 \qquad a > 0$

$y \geq 0$

solve it

$$\frac{dy}{y(b-y)} = a\,dt$$

$$\frac{1}{y(b-y)} = \frac{A}{y} + \frac{B}{b-y}$$

$$= \frac{A(b-y) + By}{y(b-y)}$$

$$A(b-y) + By = 1$$

$$-Ay + By + Ab = 1$$

$$-A + B = 0, \qquad Ab = 1$$

$$B = \frac{1}{b} \qquad A = \frac{1}{b}$$

$$\frac{1}{y(b-y)} = \frac{1}{b}\frac{1}{y} + \frac{1}{b}\frac{1}{b-y}$$

$$\frac{1}{b}\ln|y| - \frac{1}{b}\ln|b-y| = at + c$$

$$\ln\frac{|y|}{|b-y|} = abt + c'$$

$$\frac{|y|}{|b-y|} = e^{abt+c'} = De^{abt}$$

Assume $0 < y < b$

$$\frac{y}{b-y} = De^{abt}$$

$$y = bDe^{abt} - yDe^{abt}$$

$$y + yDe^{abt} = bDe^{abt}$$

$$y(t) = \frac{bDe^{abt}}{1+De^{abt}} = \frac{bD}{e^{-abt}+D}$$

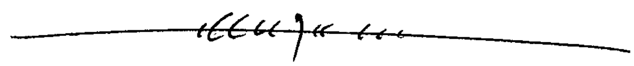
$$y(0) = \frac{bD}{1+D}$$

§ 2.4 Existence / Uniqueness

Local existence

$$\frac{dy}{dt} = g(x,y) \quad , \quad y(t_0) = y_0$$

Is there a solution which exists for t near t_0.



t_0

Global Existence

Is there a solution which exists for all t ; ~~t~~

$$-\infty < t < \infty$$

Prove global existence for Navier Stokes Equations

$\Rightarrow$ \$ 1,000,000

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Uniqueness

$$\frac{dy}{dt} = g(x, y), \qquad y(t_0) = y_0$$

Is there more than one solution (t_0, y_0) ?