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①

Logic

If A then B

Suppose A is false.

Thm says nothing

$\cancel{\frac{dy}{dx}}$ $\frac{dy}{dx} = g(x)$ ← continuous

as in chap. 1

Thm applies, so local existence

+ uniqueness

Example $\frac{dy}{dx} = \sqrt{1-y^2}$

Read book

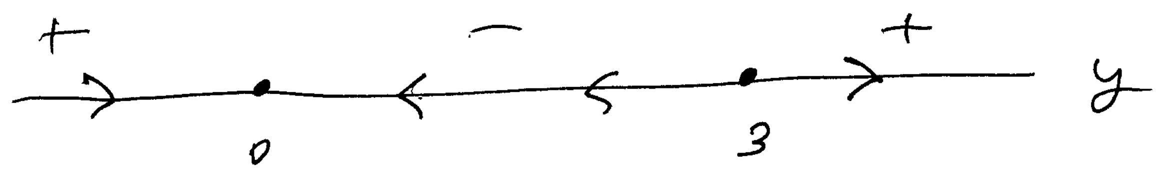
§ 2.5 phase lines

Warm up for "phase plane analysis"

Example

$$y' = y^2 - 3y = y(y-3)$$

Equilibria are $y = 0$
$y = 3$

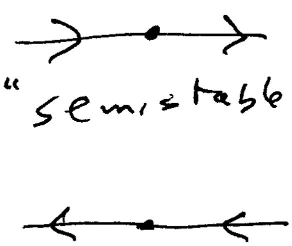


Determine signs of $y(y-3)$

0 is stable
3 is unstable

stable

unstable



"semistable"

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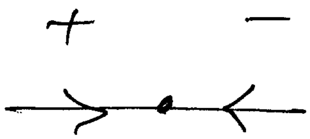
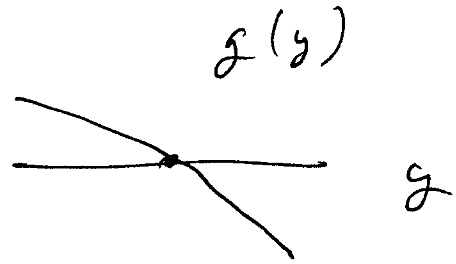
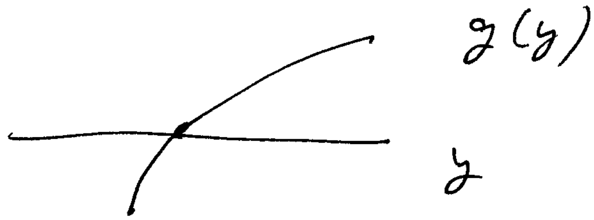
2

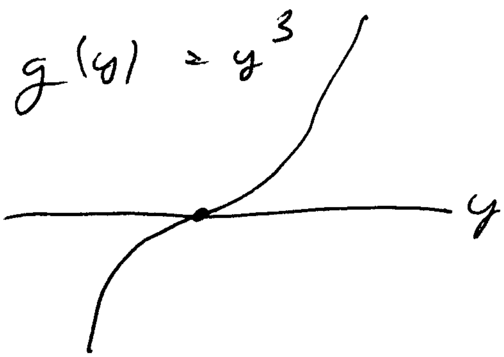






$g'(c) = 0 \Rightarrow$ nothing

$g(y) = y^3$



unstable

$$\frac{dy}{dx} = g(y)$$

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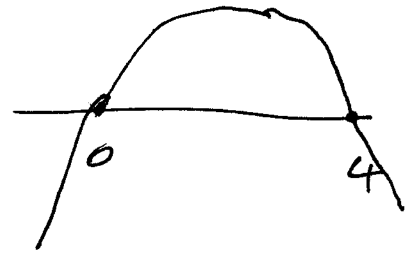
Example

$$= 8y - 2y^2$$

$$\frac{dy}{dx} = 2y(4-y) \leftarrow g(y)$$

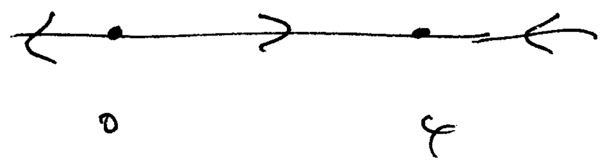
$$g' = 8 - 4y$$



$$g'(0) = 8$$

0 is unstable

$$g'(4) = -8$$



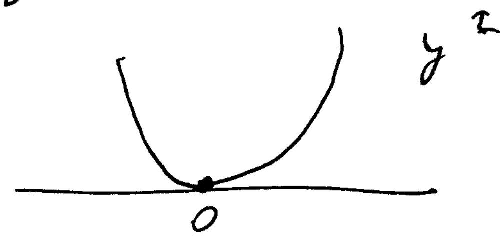
4 is stable

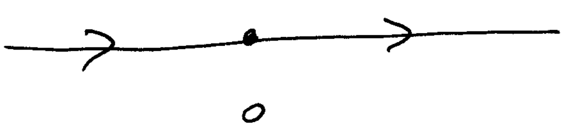
Example

$$\frac{dy}{dx} = y^2 \leftarrow g$$

0 is only equilibrium

$$g'(y) = 2y \qquad g'(0) = 0$$





semistable

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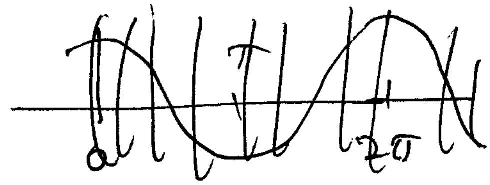
4

Example

$$\frac{dP}{dt} = \sin(\pi P) \leftarrow g(P)$$

Equilibria : $P = \ldots, -2, -1, 0, 1, 2, \ldots$

$$g'(P) = \pi \cos(\pi P)$$



$g'(0) > 0$ unstable

$g'(1) < 0$ stable

$g'(2) > 0$ unstab

$g'(3) < 0$

$2n$ is unstable

$2n+1$ is stable