

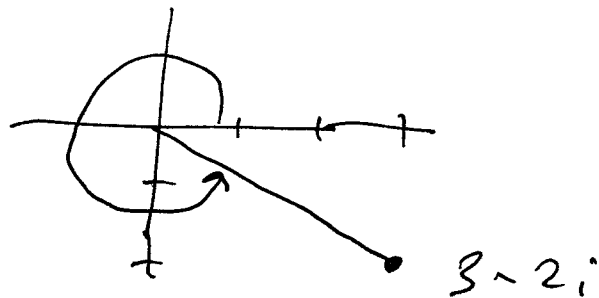
Polar coordinates

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$$x + iy$$

Plot it as (x, y)

$$3 - 2i$$



Find polar coordinates of (x, y)

$$x = r \cos \theta$$

$$y = r \sin \theta$$

$$\text{Then } x + iy = r e^{i\theta}$$

$$\tan \theta = \frac{y}{x}, \quad \theta = \tan^{-1}\left(\frac{y}{x}\right)$$

$$e^{i\theta} = \cos \theta + i \sin \theta \quad \leftarrow \begin{array}{l} 9/5 \\ 2 \end{array}$$

$$e^{-i\theta} = \cos(-\theta) + i \sin(-\theta)$$

$$e^{-i\theta} = \cos(\theta) - i \sin(\theta) \quad \leftarrow$$

Add eqs:

$$e^{i\theta} + e^{-i\theta} = 2 \cos \theta$$

Sub~~tract~~tract eqs:

$$e^{i\theta} - e^{-i\theta} = 2i \sin \theta$$

Integration

$$\int \sin \theta \, d\theta = \int \frac{e^{i\theta} - e^{-i\theta}}{2i} \, d\theta$$

$$= \frac{1}{2i} \left[\frac{1}{i} e^{i\theta} - \frac{1}{-i} e^{-i\theta} \right]$$

$$= \frac{1}{2} \left[-e^{i\theta} - e^{-i\theta} \right] = -\cos \theta$$

$$\int \sin^2 \theta \, d\theta$$

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$$\frac{1}{i} = -i$$

$$= \int \left(\frac{e^{i\theta} - e^{-i\theta}}{2i} \right)^2 d\theta$$

$$= -\frac{1}{4} \int \frac{e^{2i\theta} - 2 + e^{-2i\theta}}{1} d\theta$$

$$= -\frac{1}{4} \left[\frac{1}{2i} e^{2i\theta} - 2\theta + \frac{1}{-2i} e^{-2i\theta} \right]$$

$$= \frac{1}{2} \theta + \frac{1}{8} (i e^{2i\theta} - i e^{-2i\theta})$$

$$= \frac{1}{2} \theta + \frac{1}{8i} e^{2i\theta} - \frac{1}{8i} e^{-2i\theta}$$

$$= \frac{1}{2} \theta + \frac{-e^{2i\theta} + e^{-2i\theta}}{8i}$$

$$= \frac{1}{2} \theta + \frac{-\sin(2\theta)}{4} + C$$

$$= \frac{1}{2} \theta - \frac{1}{4} \sin(2\theta) + C$$

Find

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$$\int e^{2x} \sin(3x) dx$$

$$= \int e^{2x} \frac{e^{3ix} - e^{-3ix}}{2i} dx$$

$$= \frac{1}{2i} \int (e^{2x + 3ix} - e^{2x - 3ix}) dx$$

$$= \frac{1}{2i} \int [e^{(2+3i)x} - e^{(2-3i)x}] dx$$

$$= \frac{1}{2i} \left[\frac{1}{2+3i} e^{(2+3i)x} - \frac{1}{2-3i} e^{(2-3i)x} \right]$$

more algebra to do

$$= \frac{1}{2i} \left[\frac{1}{2+3i} e^{2x} (\cos 3x + i \sin 3x) - \frac{1}{2-3i} e^{2x} (\cos 3x - i \sin 3x) \right]$$

$$\frac{1}{2+3i} e^{(2+3i)x} = e^{2x} e^{3ix}$$

$$= e^{2x} (\cos 3x + i \sin 3x)$$

Dif Eq.

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$$\frac{dy}{dx} = x^2$$

$$\frac{dy}{dx} = x^3 + \sin y$$

$$\frac{dy}{dx} = \sin(xy)$$

$$\frac{dr}{dt} = e^{tr}$$

~~First~~ First order ↗

second order

$$\frac{d^2 y}{dx^2} + \sin x \frac{dy}{dx} + 6y = 0$$

$$F = ma$$

$$F = m \frac{d^2 x}{dt^2}$$

First order dif. eq.
is of the form

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$$\frac{dy}{dx} = g(x, y)$$

$$y(x_0) = y_0$$

e.g. $y(0) = 7$

Chap 1

$$\frac{dy}{dx} = g(x)$$

\Leftrightarrow integration $g(x)$

Chap 2

$$\frac{dy}{dx} = g(y)$$

Chap 3

$$\frac{dy}{dx} = g(x, y)$$

1) analytic

2) graphical

3) numerical

1.1 Simple DE

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Examples Find $y(x)$ such that

$$\frac{dy}{dx} = \sin x, \quad y(0) = 2$$

$$y(x) = -\cos x + C$$

$$y(0) = -\cos(0) + C = -1 + C$$

So need $-1 + C = 2$

$$C = 3$$

Solution is $y(x) = -\cos x + 3$

2. Find $y(t)$ such that

$$\frac{dy}{dt} = e^{5t}$$

$$y(1) = 10$$

$$y(t) = \frac{1}{5} e^{5t} + C$$

$$y(1) = \frac{1}{5} e^5 + C$$

so $\frac{1}{5} e^5 + C = 10$

$$C = 10 - \frac{1}{5} e^5$$

$$y(t) = \frac{1}{5} e^{5t} + 10 - \frac{1}{5} e^5$$

$\left(\begin{array}{l} 9/5 \\ 8 \end{array} \right)$