Example

$$\frac{dy}{dx} = \frac{2}{\sqrt{\pi}} e^{-x^2} \qquad (\bigstar)$$

$$F(x) = \int_0^x \frac{2}{\sqrt{\pi}} e^{-u^2}\,du$$

$$F'(x) = \frac{2}{\sqrt{\pi}} e^{-x^2}$$

$F(x)$ is a solution to $\bigstar$

$F(x)$ is called $\operatorname{erf}(x)$

I. C. $\quad y(1) = 0$

$$F(1) = \int_0^1 \frac{2}{\sqrt{\pi}} e^{-u^2}\,du$$

Want $\quad y(x) = F(x) - F(1)$

Example $\frac{1,2}{\ }$

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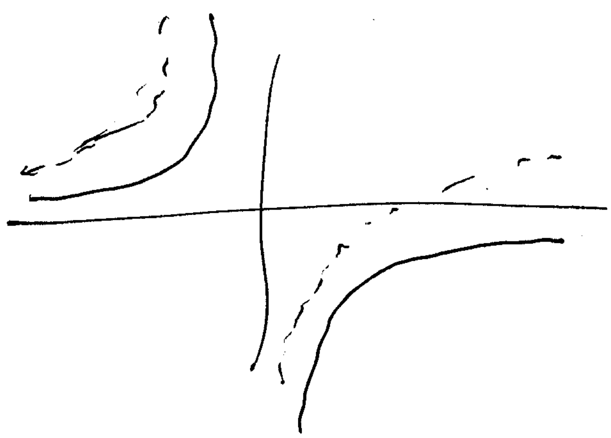
$$\frac{dy}{dx} = \frac{1}{x^2}$$

$$y(x) = \frac{-1}{x} + C$$



Monotonicity $\quad \frac{1}{x^2} > 0 \Rightarrow$ always increasing

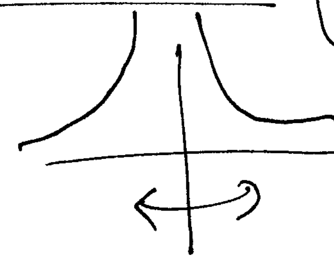
Concavity $\quad \frac{d^2y}{dx^2} = \frac{-2}{x^3}$

$x > 0 \Rightarrow \frac{d^2y}{dx^2} < 0 \Rightarrow$ concave down

$x < 0 \Rightarrow \frac{d^2y}{dx^2} > 0 \Rightarrow$ concave up

Suppose $\underline{g(x) \text{ is even}}$

$$g(-x) = g(x)$$



$$\frac{dy}{dx} = g(x)$$

suppose $y(x)$ is a solution.

Look at $y(-x)$

$$\frac{d}{dx} y(-x) = y'(-x)\,(-1)$$

$$= -g(-x)$$

$$= -g(x)$$

$$\frac{d}{dx} - y(-x) = g(x)$$

<u>Conclusion</u>: $y(x)$ is a solution

$\Rightarrow$ $-y(-x)$ is a solution