Math 250a (Kennedy) - Quiz 10 - Fall '07

(16 points) 1. (a) Solve the following linear differential equation and initial condition:

$$x\,y' + y = x\,\sin(x), \quad y(\pi) = 1$$

$y' + \frac{1}{x} y = \sin x$ $\qquad p = \frac{1}{x}$

$x y' + y = x \sin x$ $\qquad e^{\int p} = e^{\ln x} = x$

$(xy)' = x \sin x$

$xy = \int x \sin x \, dx$

$u = x \qquad dv = \sin x \, dx$

$du = dx \qquad v = -\cos x$

$xy = -x \cos x + \int \cos x \, dx$

$xy = -x \cos x + \sin x + C$

$\pi = -\pi \cos \pi + \cancel{\sin \pi}^{\,0} + C$

$\cos \pi = -1 \quad$ so $\quad C = 0$

$y = -\cos x + \frac{\sin x}{x}$

(4 points) (b) Suppose that x is time. Identify the transient and steady state parts of your solution.

As $x \to \infty$, $\frac{\sin x}{x} \to 0$

so $\frac{\sin x}{x}$ is transient part

$-\cos x$ is steady state part