Math 250a (Kennedy) - Quiz 11 - Fall '07

1. Show how to reduce $y' + \frac{y}{x} + xy^2 = 0$ to a linear differential equation. You do not need to solve the linear equation.

$n = 2 \qquad 1 - n = -1$

so let $u = y^{-1}$

$u' = -y^{-2} y' = -y^{-2}\left(-\frac{y}{x} - xy^2\right)$

$= \frac{1}{xy} + x$

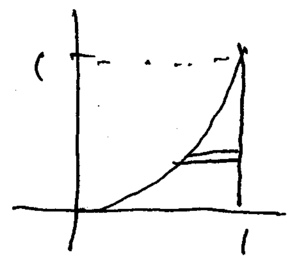
$= \frac{1}{x} u + x$

So $u' = \frac{1}{x} u + x$ OR $u' - \frac{1}{x} u = x$

2. The region bounded by $y = x^2$, the x-axis, and the vertical line $x = 1$ is rotated about the horizontal line $y = 1$. Find the resulting volume.

slice $\perp$ to y-axis

slice vol $= (1-x)\, 2\pi (1-y)\, \Delta y$



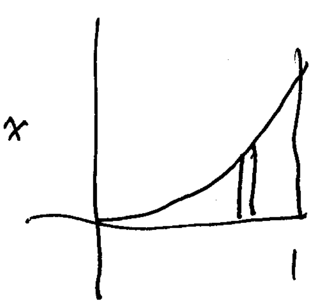
vol $= \int_0^1 (1-x)\, 2\pi (1-y)\, dy$

$= \int_0^1 (1-\sqrt{y})\, 2\pi (1-y)\, dy = \dots = \boxed{\frac{7\pi}{15}}$

OR

slice $\perp$ to x-axis

slice vol $= \pi\left[1^2 - (1-x^2)^2\right] \Delta x$



vol $= \int_0^1 \pi\left[1^2 - (1-x^2)^2\right] dx$

$= \dots = \boxed{\frac{7\pi}{15}}$