

Math 250a (Kennedy) - Quiz 3 - Fall '07

1. Find

$$\int t e^{5t} dt$$

Let $u = t$

Then $u' = 1$

$$v' = e^{5t}$$

$$v = \frac{1}{5} e^{5t}$$

$$\begin{aligned} \int t e^{5t} dt &= \frac{1}{5} t e^{5t} - \int \frac{1}{5} e^{5t} dt \\ &= \frac{1}{5} t e^{5t} - \frac{1}{25} e^{5t} + C \end{aligned}$$

$$\int x^3 \ln x dx$$

Let $u = \ln x$

then $u' = \frac{1}{x}$

$$v' = x^3$$

$$v = \frac{1}{4} x^4$$

$$\begin{aligned} \int x^3 \ln x dx &= \frac{1}{4} \ln x x^4 - \int \frac{1}{x} \frac{1}{4} x^4 dx \\ &= \frac{1}{4} x^4 \ln x - \frac{1}{16} x^4 + C \end{aligned}$$

$$\int \tan^{-1} x dx$$

Let $u = \tan^{-1} x$

then $u' = \frac{1}{1+x^2}$

$$v' = 1$$

$$v = x$$

$$\begin{aligned} \int \tan^{-1} x dx &= x \tan^{-1} x - \int \frac{x}{1+x^2} dx \\ &= x \tan^{-1} x - \frac{1}{2} \ln(1+x^2) + C \end{aligned}$$

2. Given that $\int_0^\pi x \sin x dx = \pi$. Find $\int_0^\pi x^2 \cos x dx$.

Let $u = x^2$, $v' = \cos x$

then $u' = 2x$, $v = \sin x$

$$\begin{aligned} \int_0^\pi x^2 \cos x dx &= x^2 \sin x \Big|_0^\pi - \int_0^\pi 2x \sin x dx \\ &= \pi^2 \sin 0 - 0 \sin 0 - 2\pi = \boxed{-2\pi} \end{aligned}$$