Math 250a (Kennedy) - Quiz 4 - Fall '07

1. Find the solution of each of these dif. eqs. with initial conditions.

(a)

$$\frac{df}{dt} = e^{-2t}, \quad f(0) = 0$$

$f(t) = -\frac{1}{2} e^{-2t} + C$

$f(0) = -\frac{1}{2} + C$

So $C = \frac{1}{2}$

$\boxed{f(t) = -\frac{1}{2} e^{-2t} + \frac{1}{2}}$

(b)

$$\frac{dy}{dx} = \frac{x}{1+x^2}, \quad y(0) = 2$$

Use sub $u = 1+x^2$ or guess

$\int \frac{x}{1+x^2}\, dx = \frac{1}{2} \ln\left(1+x^2\right) + C$

So $y(x) = \frac{1}{2} \ln(1+x^2) + C$

$y(0) = \frac{1}{2} \ln(1) + C = C$

So $C = 2$

$\boxed{y(x) = \frac{1}{2} \ln(1+x^2) + 2}$

2. Consider the dif. eq. and initial condition $\frac{dy}{dx} = x^2 e^{-2x^2}, \quad y(0) = 1$

(a) Determine where the solution is increasing, decreasing, concave up and concave down.

$x^2 e^{-2x^2} \geq 0$ for all x.

So $y(x)$ is always increasing.

$$\frac{d^2y}{dx^2} = 2x\, e^{-2x^2} - 4x^3 e^{-2x^2} = 2x(1-2x^2)e^{-2x^2}$$

Look at sign of $x(1-2x^2)$

	$-\frac{1}{\sqrt{2}}$		0		$\frac{1}{\sqrt{2}}$	
$+$		$-$		$+$		$-$
concave up		concave down		concave up		concave down

(b) Write down an integral that represents the solution.

$$y(x) = \int_0^x u^2 e^{-2u^2}\, du + 1$$

(c) Given that $\int_0^\infty x^2\, e^{-2x^2}\, dx = \frac{\sqrt{2\pi}}{16}$, find the horizontal asymptote of the solution as $x \to \infty$.

$$y(x) \to \int_0^\infty u^2 e^{-2u^2}\, du + 1$$

$$= 1 + \frac{\sqrt{2\pi}}{16}$$