

Math 250a (Kennedy) - Quiz 4 - Fall '07

1. Find the solution of each of these dif. eqs. with initial conditions.

(a)

$$\frac{df}{dt} = e^{-2t}, \quad f(0) = 0$$

$$f(t) = -\frac{1}{2} e^{-2t} + C$$

$$f(0) = -\frac{1}{2} + C$$

$$\text{So } C = \frac{1}{2}$$

$$\boxed{f(t) = -\frac{1}{2} e^{-2t} + \frac{1}{2}}$$

(b)

$$\frac{dy}{dx} = \frac{x}{1+x^2}, \quad y(0) = 2$$

Use sub $u = 1+x^2$ or guess

$$\int \frac{x}{1+x^2} dx = \frac{1}{2} \ln(1+x^2) + C$$

$$\text{So } y(x) = \frac{1}{2} \ln(1+x^2) + C$$

$$y(0) = \frac{1}{2} \ln(1) + C = \cancel{0} + C$$

$$\text{So } C = 2$$

$$\boxed{y(x) = \frac{1}{2} \ln(1+x^2) + 2}$$

2. Consider the dif. eq. and initial condition $\frac{dy}{dx} = x^2 e^{-2x^2}$, $y(0) = 1$

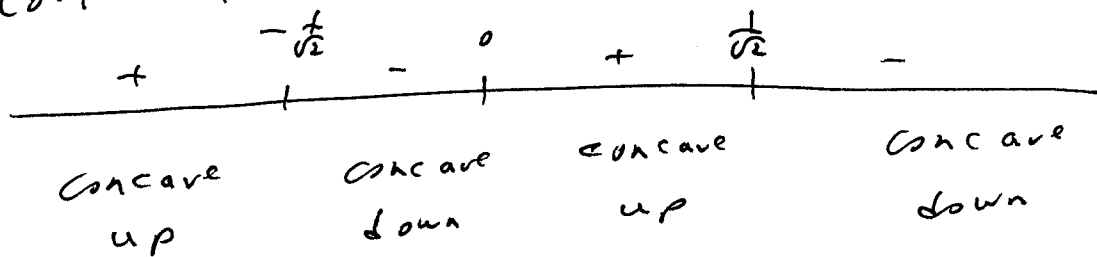
(a) Determine where the solution is increasing, decreasing, concave up and concave down.

$$x^2 e^{-2x^2} \geq 0 \quad \text{for all } x.$$

So $y(x)$ is always increasing.

$$\frac{d^2 y}{dx^2} = 2x e^{-2x^2} - 4x^3 e^{-2x^2} = 2x(1 - 2x^2) e^{-2x^2}$$

Look at sign of $x(1 - 2x^2)$



(b) Write down an integral that represents the solution.

$$y(x) = \int_0^x u^2 e^{-2u^2} du + 1$$

(c) Given that $\int_0^\infty x^2 e^{-2x^2} dx = \frac{\sqrt{2\pi}}{16}$, find the horizontal asymptote of the solution as $x \rightarrow \infty$.

$$\begin{aligned} y(x) &\rightarrow \int_0^\infty u^2 e^{-2u^2} du + 1 \\ &= 1 + \frac{\sqrt{2\pi}}{16} \end{aligned}$$