

Math 250a (Kennedy) - Quiz 4 - Fall '07

1. $P(t)$ is the population of a bacteria culture. It satisfies $\frac{dP}{dt} = kP$ for some constant k . After 20 hours the population is 3 times its original size.

(a) Find k .

$$P(t) = P(0) e^{kt}$$

$$P(20) = 3 P(0)$$

$$e^{20k} = 3, \quad k = \frac{\ln 3}{20}$$

(b) Find the doubling time.

$$e^{kt} = 2 \Rightarrow t = \frac{\ln 2}{k}$$

$$t = \frac{20 \ln 2}{\ln 3}$$

2. Use partial fractions to do the following integral. Show your work.

$$\int \frac{x^2 + x + 4}{4x + x^3} dx$$

$$\frac{x^2 + x + 4}{x(4 + x^2)} = \frac{A}{x} + \frac{Bx + C}{x^2 + 4}$$

$$x^2 + x + 4 = A(x^2 + 4) + x(Bx + C)$$

$$\left. \begin{array}{l} 1 = A + B \\ 1 = C \\ 4 = 4A \end{array} \right\} \Rightarrow \begin{array}{l} A = 1 \\ B = 0 \\ C = 1 \end{array}$$

$$\text{Get } \int \left(\frac{1}{x} + \frac{1}{x^2 + 4} \right) dx$$

$$= \ln|x| + \int \frac{2 du}{4u^2 + 4}$$

$$= \ln|x| + \frac{1}{2} \tan^{-1}\left(\frac{x}{2}\right) + C$$

$$\left. \begin{array}{l} \text{Let} \\ 4u^2 + 4 = x^2 + 4 \\ 2u = x \\ 2du = dx \end{array} \right\}$$