Math 250a (Kennedy) - Quiz 6 - Fall '07

1. Find the solution of the differential equation $y' = y^2$ that satisifes $y(0) = 1$.

$\frac{dy}{dx} = y^2$ $\qquad \frac{dy}{y^2} = dx$

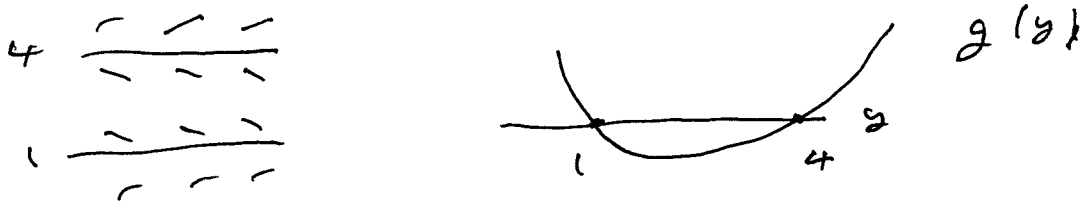
$-\frac{1}{y} = x + C$

$y(0) = 1 \Rightarrow \quad -1 = C$

$-\frac{1}{y} = x - 1$

$y = \frac{1}{1-x}$

2. Give an example of a differential equation such that $y(x) = 1$ is a stable equilibrium solution, $y(x) = 4$ is an unstable equilibrium solution and there are no other equilibrium solutions.



$\frac{dy}{dx} = (y-1)(y-4)$

3. The logistic equation is $y' = ay(b - y)$ with $a, b > 0$. Suppose the intial population is greater than the carrying capacity, i.e., $y(0) > b$. Is $y(x)$ ever less than b? Explain your answer without solving the equation.

Uniqueness theorem applies. So solutions cannot cross. $y(x) = b$ is a solution. So a solution that starts above this horizontal line must stay above it. So answer is NO.