

Math 250a (Kennedy) - Quiz 7 - Fall '07

1. Consider the differential equation: $y' = (y - 5)(y - a)$, where a is a parameter with no restrictions. Clearly $y = 5$ and $y = a$ are equilibrium solutions. Determine when each is stable, unstable and semistable. You can give your answer in the form of sentences or a bifurcation diagram.

$$g(y) = (y-5)(y-a) = y^2 - 5y - ay + 5a$$

$$\frac{dg}{dy} = 2y - 5 - a$$

$$\text{At } y=a, \frac{dg}{dy} = a-5. \quad \text{At } y=5, \frac{dg}{dy} = 5-a$$

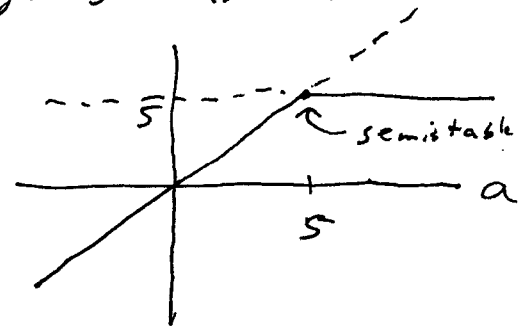
So if $a > 5$, $y=a$ is unstable, $y=5$ is stable

If $a < 5$, $y=a$ is stable, $y=5$ is unstable.

$$\text{At } a=5, g(y) = (y-5)^2$$

which is always ≥ 0 .

So $y=5$ is semistable



2. Let $y(x)$ be the solution of $y' = x/y$ with the initial condition $y(1) = 2$. For each of the following show your work and **reasoning**. You can use the slope field to check your answer, but correct answers only get no points.

(a) Determine if y is increasing or decreasing at $x = 1$ and explain why.

$$y'(1) = \frac{1}{2} > 0, \text{ so it's increasing.}$$

(b) Determine if y is concave up or down at $x = 1$ and explain why.

$$y'' = \frac{y - xy'}{y^2} = \frac{y - x \frac{x}{y}}{y^2}$$

$$\text{So } y''(1) = \frac{2 - \frac{1}{2}}{4} > 0$$

So it's concave up at $x=1$.