Math 250b (Spring '08) - Homework 11

1. Here is a "two compartment model" for the movement of a drug in the body. Compartment P is the blood plasma, compartment T is tissue. From the blood (P) the drug can move to the tissue (T) or be excreted. From the tissue it can only move back to the blood. The volume of the plasma is 8 liters, the tissue is 4 liters. Fluid with drug in it flows from the blood plasma to the tissue at the rate of 5 liters/hr, and from the tissue to the blood plasma at the same rate. Drug is excreted via fluid leaving the blood stream at the rate of 4 liters/hr. (Fluid without drug is added to the blood plasma so that the volume of blood plasma says constant.)

(a) Let $C_P(t)$ and $C_T(t)$ be the concentrations of the drug in the blood and tissue. Write down a system of differential equations for these two functions.

(b) We inject $0.5g$ of the drug by IV, so it instantaneously enters the blood. Find $C_P(t)$ and $C_T(t)$. (Think of the injection as an initial condition. At $t = 0$ the blood has 0.5 g of drug, the tissue has none.)

This problem was based on a pharmacology paper: "Usefulness of the Two-Compartment Open Model in Pharmacokinetics," by Carl M. Metzler, Journal of the American Statistical Association, Vol. 66, No. 333 (Mar., 1971), pp. 49-53. There is a link to it on the homework web page. In particular, I tried to get "realistic" constants out of the paper. So the numbers are not rigged to avoid square roots.