Math 250b (Spring '08) - Homework 13 - Part II

1. Consider

$$\begin{aligned} x' &= (y+1)(y-c) \\ y' &= e^x - 1 \end{aligned}$$

(a) Find the equilibrium points. (They will of course depend on c.)
(b) Use linearization to say what you can about the nature of the equilibria. (This will also depend on c.)
(c) There is a value of c at which there is a qualitative change in the behavior of the system. What is it?

2. Two populations $x(t)$ and $y(t)$ compete for the same resource. We model them by

$$\begin{aligned} x' &= x(5 - cx - y) \\ y' &= y(5 - cy - x) \end{aligned}$$

(a) Find the equilibrium points.
(b) Use linearization to say what you can about the nature of the equilibria.
(c) There is a value of c at which there is a qualitative change in the behavior of the system. What is the value?
(d) Explain this qualitative change in terms of the populations being modelled.

3. A non-linear pendulum with friction satifies the equation

$$x'' + kx' + 3\sin(x) = 0$$

where $k \geq 0$.
(a) Find the equilibrium points for the corresponding first order system.
(b) Use linearization to say what you can about the nature of the equilibria.
(c) There is a value of k at which there is a qualitative change in the behavior of the system. What is the value?