

### Sample Exam 1 - Math 464 - Spring 07 -Kennedy

*The questions on this sample exam are meant to be representative of the questions that will be on the exam. However, this sample exam is a bit too long for a 50 minute exam.*

1. In the following two questions  $A$  and  $B$  are events.
  - (a) Suppose  $P(A) = 0.5$  and  $P(B) = 0.4$ . Find  $P(A \setminus B)$  if  $A$  and  $B$  are independent. ( $A \setminus B$  is  $A \cap B^c$ .)
  - (b) Suppose  $P(A) = 0.6$ ,  $P(B) = 0.3$  and  $P(A \cup B) = 0.8$ . Find the probability of  $A$  given  $B$ , i.e.,  $P(A|B)$ .
2. Each day a weatherman makes one of three predictions: “rain”, “no rain”, or “possibility of rain.” The percentages of the time he makes each prediction are 10%,75%,and 15% respectively. If the weather forecast is for “rain,” the probability it will rain is 70%. If the forecast is “no rain,” the probability it will rain is 20%. If the forecast is for “possibility of rain”, the probability of rain is 50%.
  - (a) Find the percentage of days on which it rains.
  - (b) Suppose it did not rain yesterday. What is the probability the forecast for yesterday was for “no rain?”
3. (a) Let  $X$  be a geometric RV with mean 2. Compute  $E[e^{-X}]$ .  
(b) Now suppose I flip a fair coin  $n$  times. Let  $X$  be the number of heads. Find  $E[3X - 2]$  and  $E[2X^2 + 1]$ .
4. The number of customers that arrive at a bank during a one minute interval is  $X$ . We assume it has a Poisson distribution with mean 2. So the pmf is given by the formula in the table with  $\lambda = 2$ . We assume the customers are independent and for each customer the probability they make a deposit is  $1/4$ . Let  $Y$  be the number of customers who arrived in the one minute interval who make deposits. Find the mean of  $Y$ .
5. I have 10 one-dollar bills, 6 five-dollar bills and 2 ten-dollar bills.
  - (a) If I randomly arrange the bills in a row, what is the probability that as I look at them from left to right I first see all the one-dollar bills, then all the five-dollar bills and finally the ten-dollar bills?
  - (b) In how many ways can I give all 18 bills to my three best friends? (Assume that bills with the same value are identical, and there are no constraints on how many bills each friend gets.)
6. An urn contains  $n$  balls numbered 1 to  $n$ . I draw three balls, one at a time without replacement.

- (a) What is the probability the three drawn are all less than or equal to 5?  
(Assume that  $n > 5$ .)
- (b) What is the probability the three I draw are of the form  $k, k + 1, k + 2$ , i.e., the second one drawn is one more than the first drawn and the third one drawn is one more than the second one drawn?
- (c) What is the probability the three drawn are in increasing order, i.e., if the balls drawn are  $i, j, k$  in that order, what is the probability  $i < j < k$ ?