**Sample Exam 2 - Math 464/564 - Spring 07 -Kennedy**

**Show your work! Correct answers with no work get no points.**

1. Let $X$ be a continuous random variable that is uniformly distributed on $[-1, 1]$. Let $Y = X^3$.

(a) Find the mean and variance of $Y$.
(b) Find the probability density function (pdf) of $Y$.

2. Let $X$ and $Y$ be continuous random variables with joint pdf

$$f_{X,Y}(x, y) = \frac{3}{2}(x^2 + y^2), \quad 0 \le x \le 1, 0 \le y \le 1$$

Outside of $0 \le x \le 1, 0 \le y \le 1$, $f_{X,Y}(x, y) = 0$.

(a) Are $X$ and $Y$ independent?
(b) Compute $P(X \le Y)$ and $P(2X \le Y)$.

3. Let $n, m$ be positive integers and let $0 < p < 1$, $0 < q < 1$. Let $X$ and $Y$ be discrete random variables with joint pmf

$$f_{X,Y}(j, k) = \binom{n}{j}\binom{m}{k} p^j (1-p)^{n-j} q^k (1-q)^{m-k}$$

where $j = 0, 1, 2, \cdots, n$ and $k = 0, 1, 2, \cdots, m$.

(a) Are $X$ and $Y$ independent?
(b) Find the mean and variance of $Z = X + Y$.

4. Let $X$ be a Poisson random variable with parameter $\lambda$. Let $Y = X + 1$.

(a) Find the moment generating function of $Y$.
(b) Use your moment generating function to compute the mean and variance of $Y$. (Note that you can check your work by computing the mean and variance of $Y$ from the mean and variance of $X$.)