Sample Final Exam - Math 464/564 - Spring 07 -Kennedy

1. I flip a fair coin. If it is heads I roll a four-sided die, and if it is tails I roll a six-sided die.
(a) What is the probability the die roll is 3.
(b) If the die roll is 3, what is the probability the coin flip showed heads?

2. I flip a fair coin until I get heads.
(a) Find the probability it takes at most 3 flips (including the flip that comes up heads).
(b) Let X be the number of flips it takes (including the flip that gives heads). Compute $E[X|X \leq 3]$, the expected value of X given that X is at most 3.

3. We roll two ordinary six-sided dice. Let X be the number of dice showing a 1 and Y the number of dice showing a 2. So each of X and Y can be $0, 1$ or 2.
(a) Find the joint density of X, Y.
(b) Are X and Y independent. Justify your answer.
(c) Compute $E(XY)$.

4. Let X have the gamma distribution with $\lambda = 1/2$ and $w = 1/2$. Find the p.d.f. of $Y = \sqrt{X}$.

5. Let X and Y be continuous random variables with joint density

$$f(x, y) = \begin{cases} \frac{6}{7}(x+y)^2, & \text{if } 0 \leq x \leq 1,\ 0 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

(a) Find the marginal densities of X and Y.
(b) Are X and Y independent ?

6. A point (X, Y) is chosen at random from the triangle with vertices $(0, 0)$, $(1, 0)$ and $(0, 1)$. (In other words, the joint distribution of (X, Y) is uniform on this triangle.) Let $Z = X + Y$.
(a) Find $P(Z < 1/2)$
(b) Find the pdf of Z.

7. (a) Suppose X is a random variable whose moment generating function $M(t)$ satisfies the equation $M(t) = M(-t)$. Show that the mean of X is 0.
(b) Now suppose the moment generating function $M(t)$ satisfies the equation $M(t) = e^t M(-t)$. Find the mean of X.

8. The joint density of X and Y is

$$f_{X,Y}(x,y) = e^{-x-y}, \qquad 0 \leq x, y < \infty$$

Let

$$U = X + Y, \quad V = X - Y$$

(a) Find the joint pdf of U, V.
(b) Compute $E[UV]$.
(c) Show that U and V are not independent random variables. (There are many ways to do this, some are easier than others.)

9. Let X and Y be independent random variables, each of which has the standard normal density. Let $Z = 2X + Y - 5$.
(a) Find the pdf of Z.
(b) Find $E[Z|X]$.

10. Let $X_1, X_2, \cdots X_n$ be independent continuous random variables. They are identically distributed, i.e., they have the same distribution. Suppose that $EX_i = 1$ and $EX_i^2 = 3$. Define

$$X = \sum_{i=1}^{n} X_i \tag{2}$$

(a) Find the mean and variance of X.
(b) If Z has a standard normal distribution, then $P(|Z| < 1.96) = 0.95$ and $P(|Z| < 1.64) = 0.90$. Find c so that for large n, $P(X - n < c)$ is approximately 0.95. Your answer should depend on n. (Note that it is $X - n$ and not $|X - n|$.)